Critical behavior of a parity conserving cellular automaton with spatial disorder

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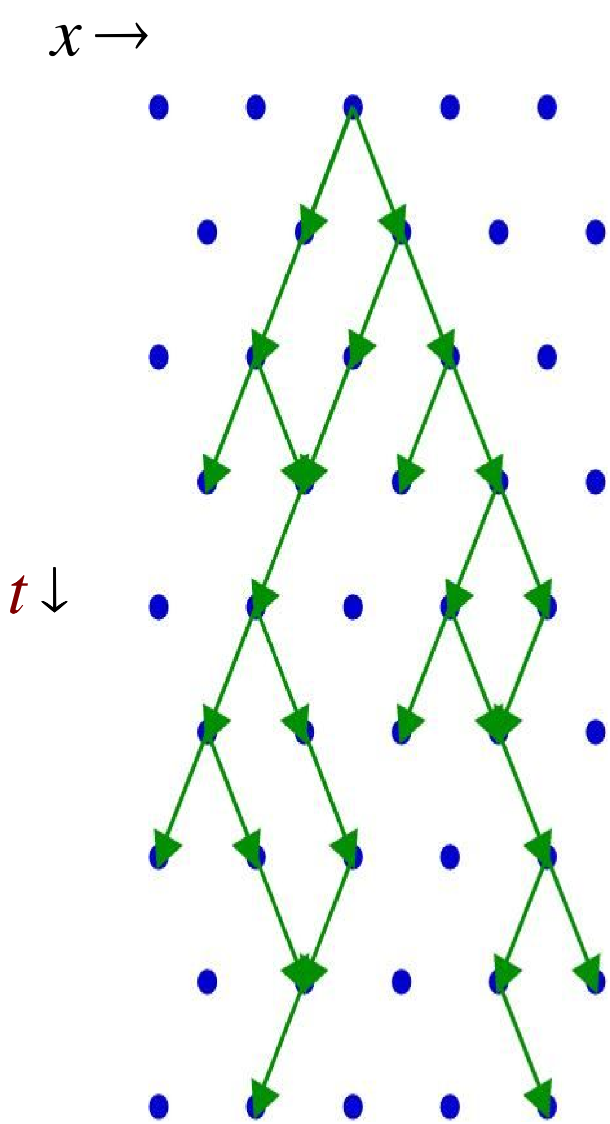
Motivation: finding nonequilibrium critical phase transition universality class candidate for experimental observation

Branching and annihilating random walks (BARW)

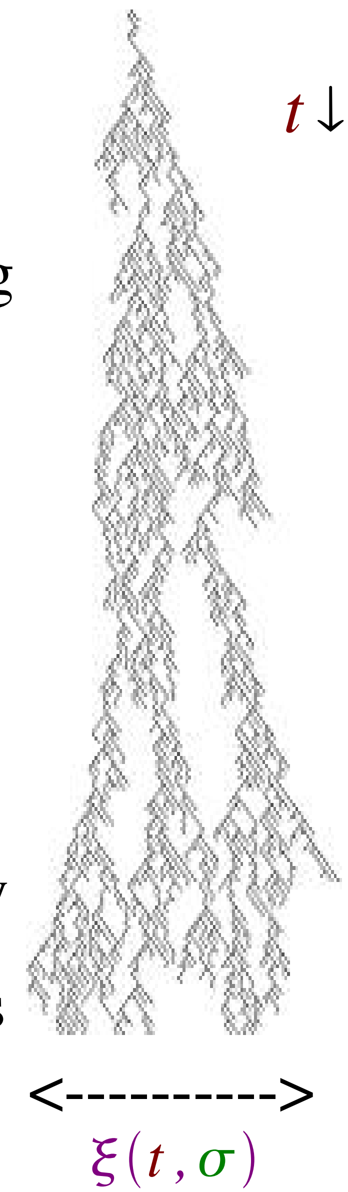


- A $\xrightarrow{\sigma}$ (n+1)A,   2A -> A,   A -> 0
  <-- 1+1 d realization -->
- For n=1 directed percolation (DP), contact process, epidemic spreading ~ BARW1 ~ BARWodd (no A->0)
- Phase transition with DP class universal behavior:

$$\rho_\infty \propto (\sigma - \sigma_c)^{\beta} \qquad \rho(t) \propto t^{-\alpha}$$

$$\xi_\infty \propto (\sigma - \sigma_c)^{-\nu} \qquad R \propto \xi \propto t^{z/2}$$

- Well known exponents, field theory
- Robust universality class, still lacks experimental realization
  <- sensitivity to disorder

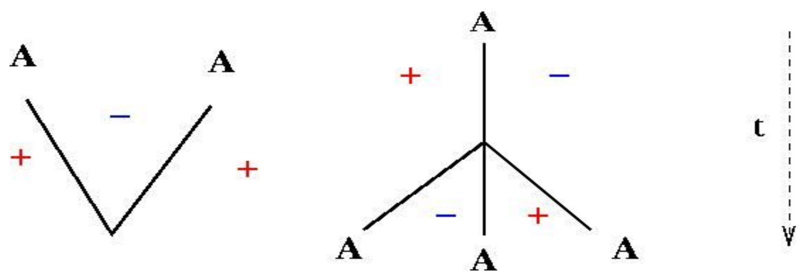


Annihilating random walks (ARW)

- A + A -> A reaction (~ A + A -> 0)
- Mean-field description: $d\rho(t,x)/dt = D\Delta\rho(t,x) - k\rho^2(t,x)$
  asymptotic solution : $\rho(t) \sim 1/t$
- d <= 2 spatial dimensions the fluctuations are relevant:
  d = 1 : $\rho(t) \sim 1/\sqrt{t}$
  d = 2 : $\rho(t) \sim \ln(t)/t$ "upper critical dimension": $d_c = 2$
- Experimental verification in 1d TMMC:
  R. Kroon, H. Fleurent, R. Sprik, PRE 47 (1993) 2462.

The parity conserving universality class (PC) BARW2, BARWe

- First seen in 1d stochastic CA by Grassberger 1984
- Nonequilibrium kinetic Ising modell (NEKIM)
  (T=0 spin flipp + T>0 spin flipp) *N. Menyhárd, J. Phys. A 1994.*



Z_2 symmetry, domain walls --> particles: AA-> 0 A-> 3A
parity conservation, non-DP class : (AA-> 0, A-> 2A)

- BARW2 or BARWe models Takayasu&Tretyakov, Jensen ....
- Field theoretical description Cardy & Tauber ('98) $d_c = 4/3$
  NPRG by Canet et al. 2005

Knowledge about the effects of disorder (spatial quenched)

- Harris stability criterion: $d\nu > 2$
  PC class: $d=1, \nu=1.857(1)$ -> **disorder relevant**

- Real space renormalization study of BARW2
  (*Hooyberhs et al. 1999*) : **No strong disorder fixed point**

- Inactive phase (ARW) :
  - Barriers and traps to diffusion -> subdiffusive behavior below a critical temperature, **slower nonuniversal power law**
    Schütz & Mussawisade 1998 (exact calculation)

  - Random reaction probabilities -> **marginal perturbation**
    Doussal & Monthous 1999 (real space renormalization)

NEKIMCA model algorithm

(*Menyhárd & Ódor, JPA 1995*)

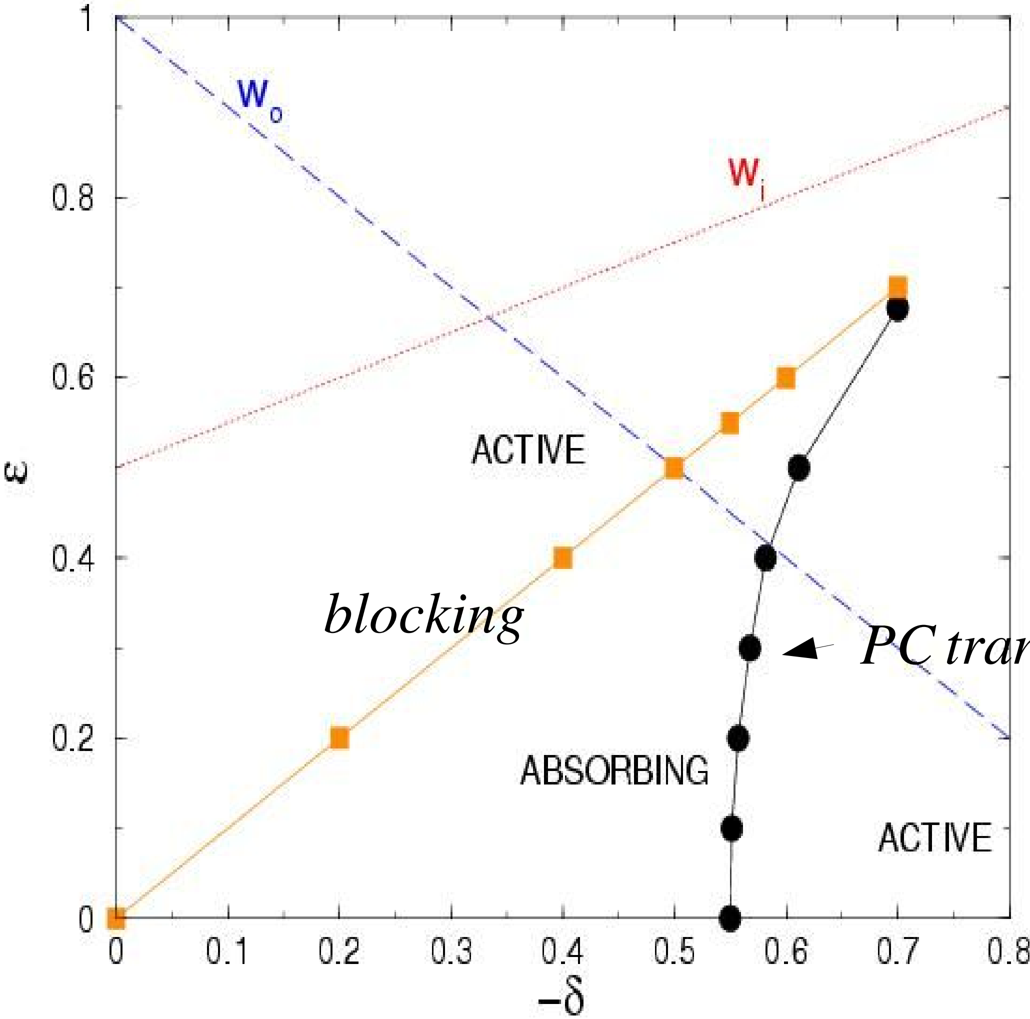
- Diffusion: $\uparrow\uparrow\cdot\downarrow \xrightarrow{w_i} \uparrow\cdot\downarrow\downarrow$ (• = A)
- Annihilation: $\uparrow\cdot\downarrow\cdot\uparrow \xrightarrow{w_o} \uparrow\uparrow\uparrow$
- Branching: $\uparrow\uparrow\cdot\downarrow\downarrow \xrightarrow{w_i^2} \uparrow\cdot\downarrow\cdot\uparrow\cdot\downarrow$

  where: $w_i=(1-\delta)/2$ $w_o=(1+\delta)$

  The branching is the result of two overlspping diffusion steps in this syncronous SCA !

- Multispin (bit) coding, 32 or 64 samples evolve at once with the same random number sequence.

Results: Phase diagram



- Uniformly distributed disorder:

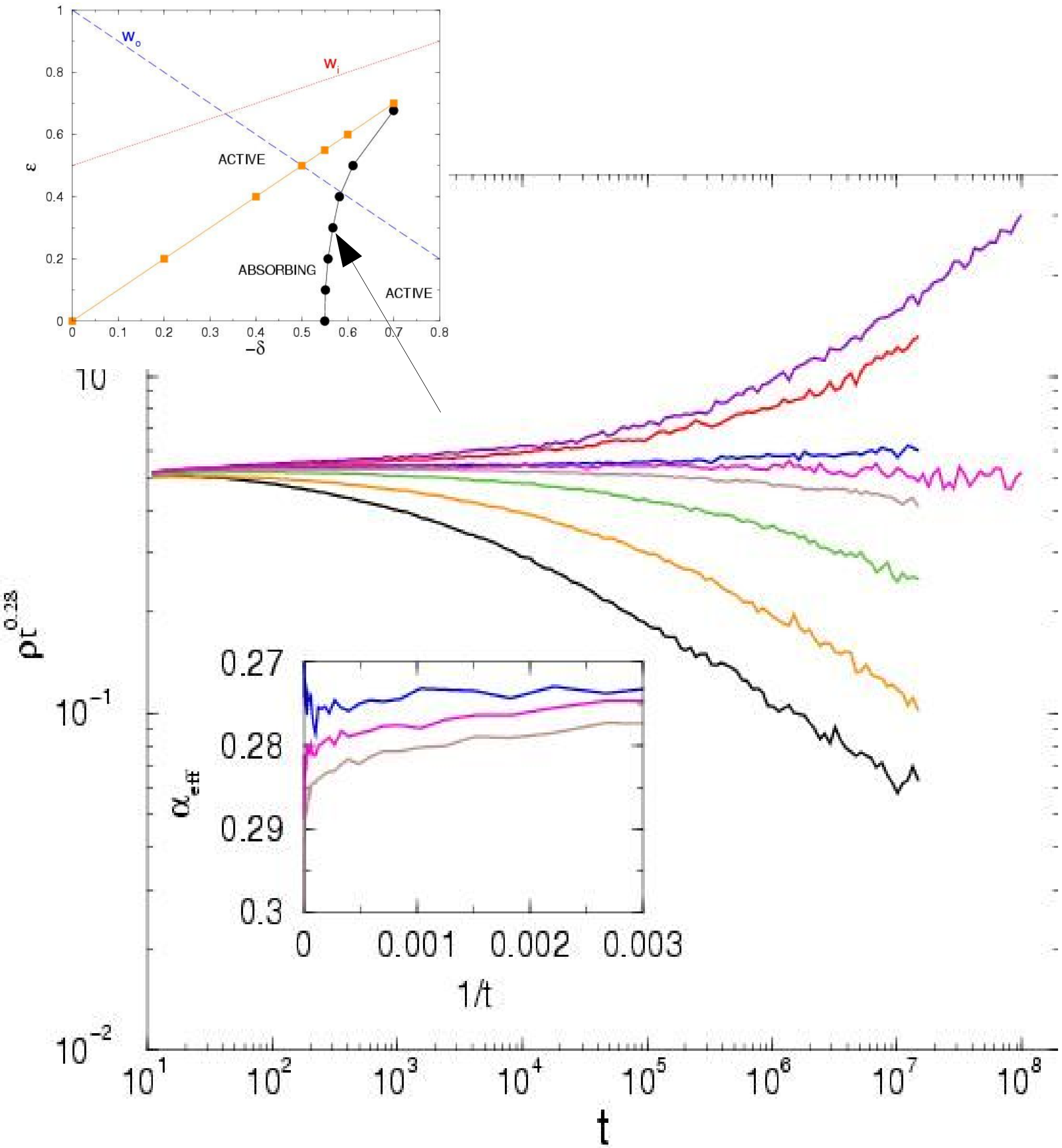
$$x(j) < q_i(j) = w_i + \chi(j) \qquad x \in (0,1)$$

$$y(j) < q_o(j) = w_o + \chi(j) \qquad y \in (0,1)$$

$$\chi \in (-\epsilon, \epsilon)$$

- Disorder increases the size of the absorbing phase
- Diffusion blocks where: $q_o(j) > 1$
- In odd parity blocks particles survive "blocking" transition to a new active phase

Scaling behavior on the PC class transition line I. (weak disorder ε < ~0.4)



- Kink density decay from homogenous, random inintial state, $L=10^5$, $t_{max}=10^8$ MCS

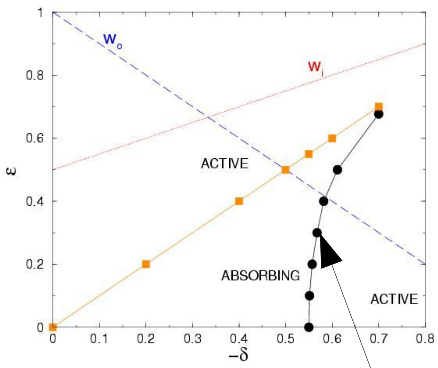
$$\rho \propto t^{-\alpha}$$

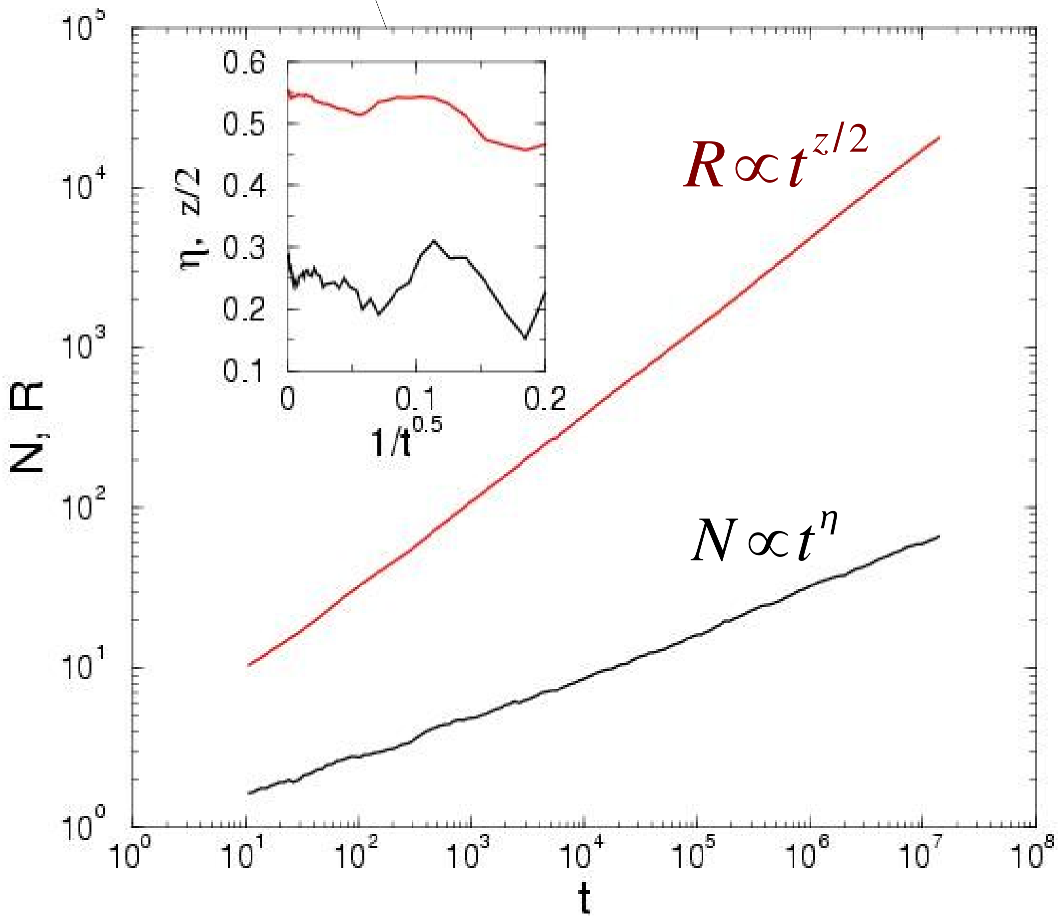
- PC class ($\alpha = 0.28(1)$) for weak disorder ($\varepsilon = 0.0, 0.1, 0.2, 0.3, 0.4$)
- Effective exponent:

$$\alpha_{eff} = \frac{-\partial \ln(\rho)}{\partial \ln(t)}$$

$\epsilon = 0.3, -\delta = 0.571, 0.57, 0.568, 0.5675, 0.567, 0.565, 0.56, 0.55$

Scaling behavior on the PC-s class line II. (weak disorder ε< ~0.4)





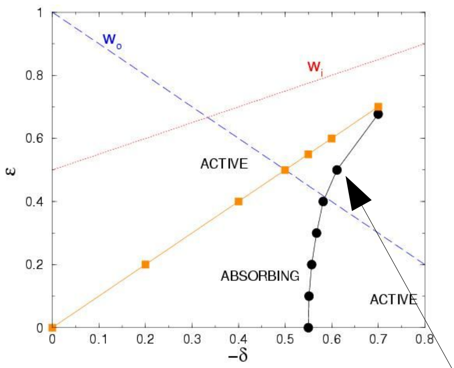
$\epsilon = 0.3, -\delta_c = 0.5679$

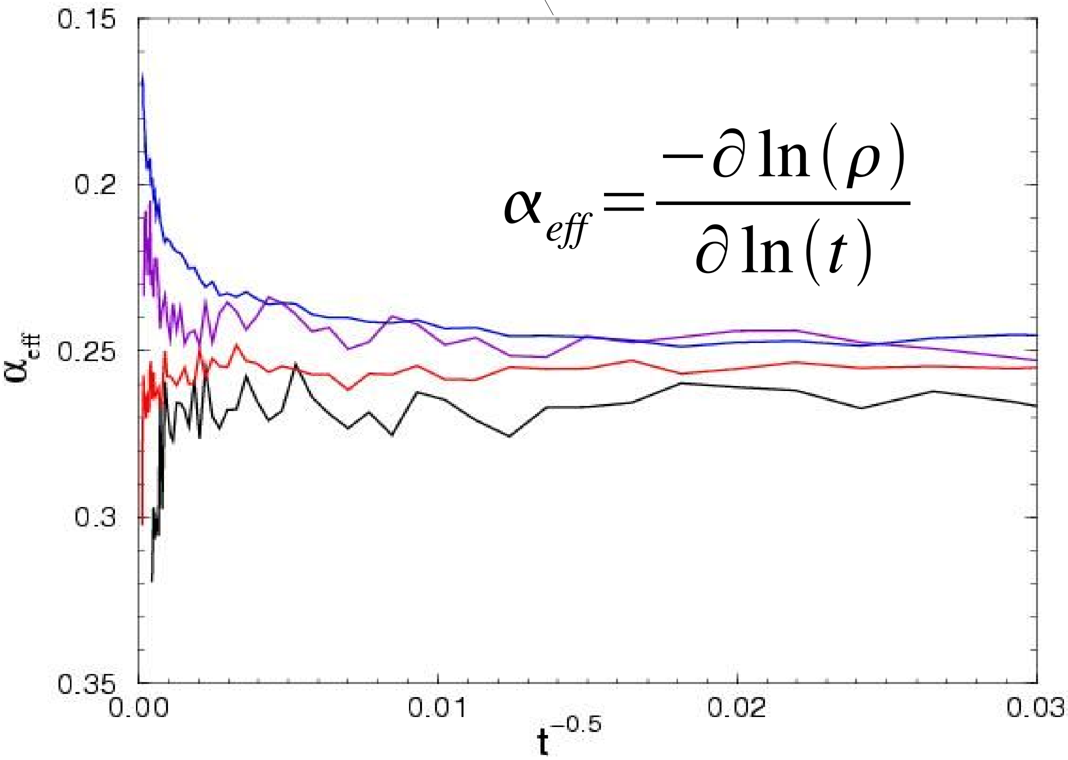
- Cluster evolution from a single kink, (odd sector), $L = 10^5$, $t_{max} = 10^7$ MCS
- PC class:
  $\eta = 0.28(1)$, $z = 1.11(1)$
- Effective exponents:

$$\eta_{eff} = \frac{\partial \ln(N)}{\partial \ln(t)} \quad z/2_{eff} = \frac{\partial \ln(R)}{\partial \ln(t)}$$

Scaling behavior on the PC line III.

(strong disorder ε > ~0.4)





$$\alpha_{eff} = \frac{-\partial \ln(\rho)}{\partial \ln(t)}$$

$\epsilon = 0.5, -\delta = 0.613, 0.612, 0.611, 0.609$

- Kink density decay from homogenous, random inintial state, L =10^5, t_{max} = 10^8 MCS
  Blocking of annihilation -> continuously changing α
  (ε = 0.5, 0.67)
- Cluster exponents z,η decrease in this region.
- Rare region argument (ala Noest)

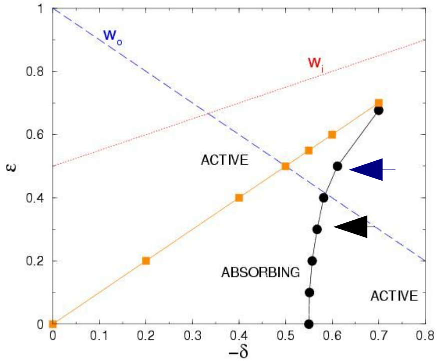
$$p_a \propto \exp(-q l_a), \quad (14)$$

$$\rho(t) \propto \int dl_a l_a p_a \exp[-t/\tau(l_a)], \quad (15)$$

$$\tau(l_a) \propto \exp(a l_a) \quad (16)$$

$$\rho(t) \propto t^{-p_a/a}. \quad (17)$$

Scaling behavior below the PC line
(active phase)



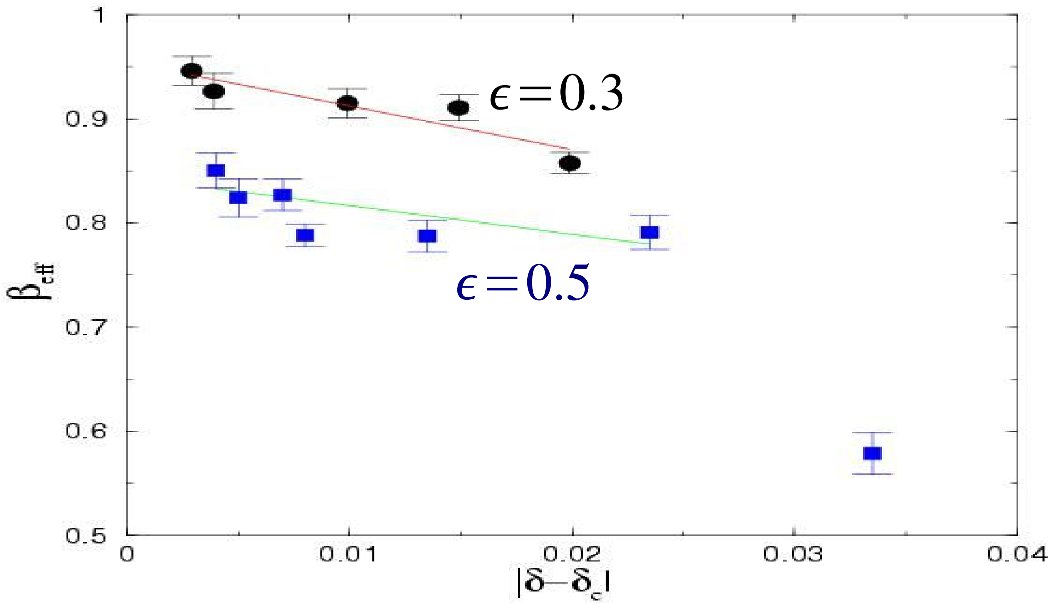


TABLE I. Numerical results for the disordered PC class transition line.

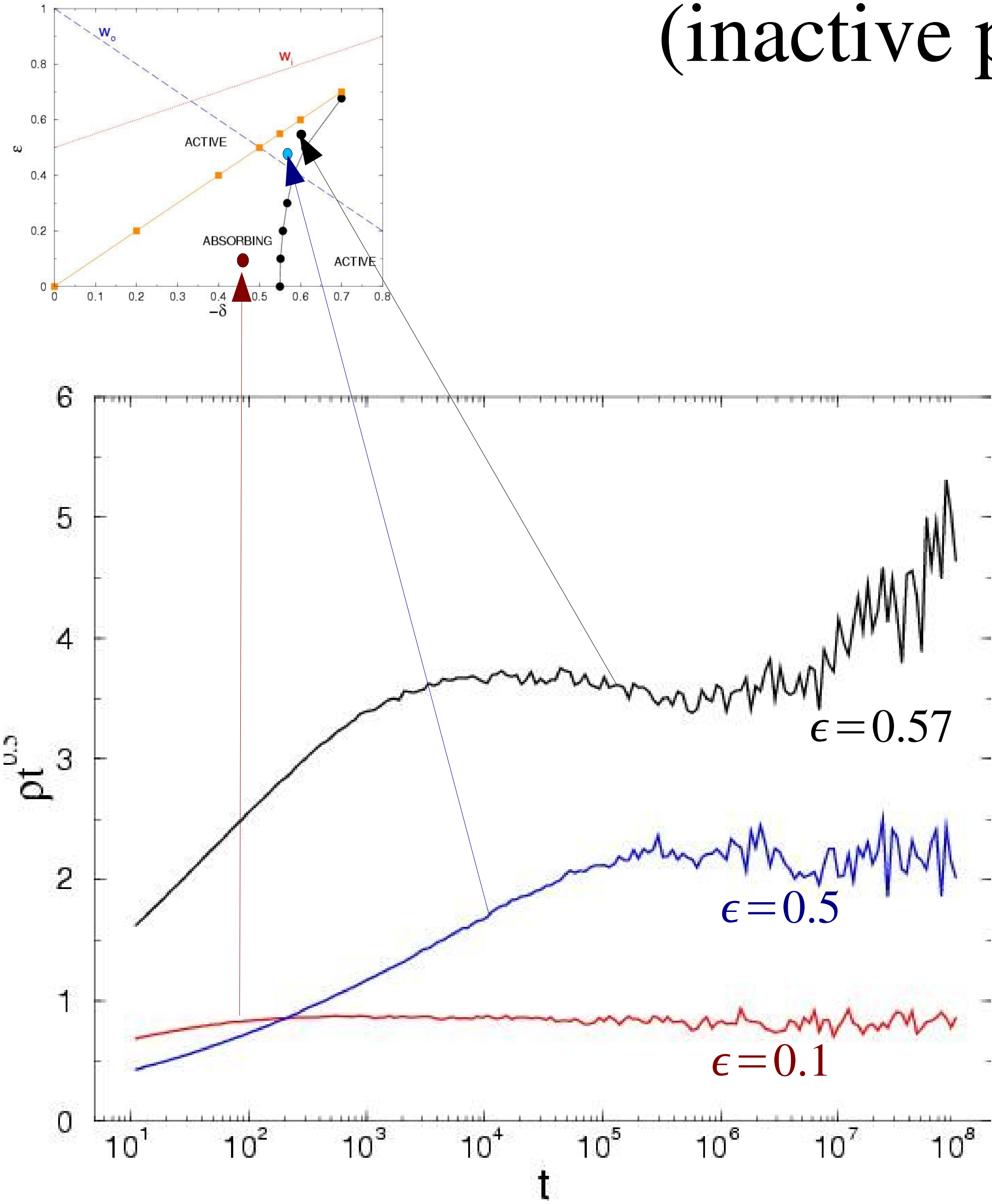
ϵ	$-\delta_c$	α	β	z	η
0.0	0.550(1)	0.280(6)			
0.1	0.5513(5)	0.280(6)			
0.2	0.557(1)	0.280(6)			
0.3	0.5676(1)	0.280(5)	0.95(1)	1.11(1)	0.285(5)
0.4	0.5849(1)	0.265(10)			
0.5	0.6115(4)	0.25(1)	0.84(2)	1.0(1)	0.252(6)
0.6767(2)	0.7	0.22(1)	0.80(2)		

- Steady state density effective exponent:

$$\beta_{eff} = \frac{-\partial \ln(\rho_\infty)}{\partial \ln(\epsilon)}$$

- PC class β = 0.95(2)
  ε < 0.4-re
- Continuously changing β , for strong disorder: ε > ~ 0.4
  Cluster exponents z, η decrease as well.

Scaling behavior above the PC line
(inactive phase)

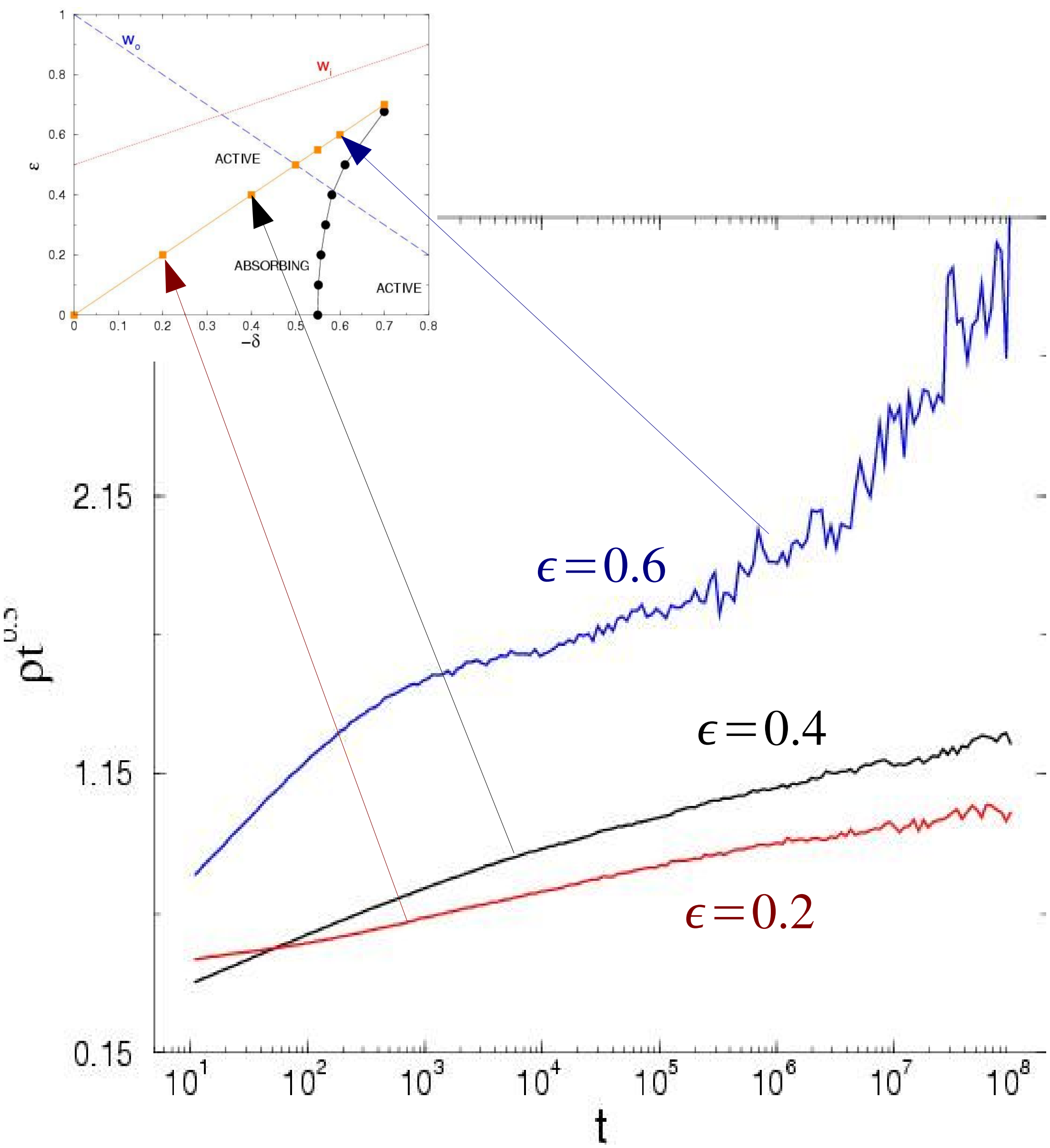


- Kink density decay from homogenous, random inintial state, L=10^5, t_{max}=10^8 MCS

$$\rho \propto t^{-\alpha}$$

- ARW class behavior α = 0.50(1) for small ε
- Increasing α just below the blocking transition line
- Agreement with Schütz's results

Scaling behavior on the blocking transition line (diffusion blocks) I.

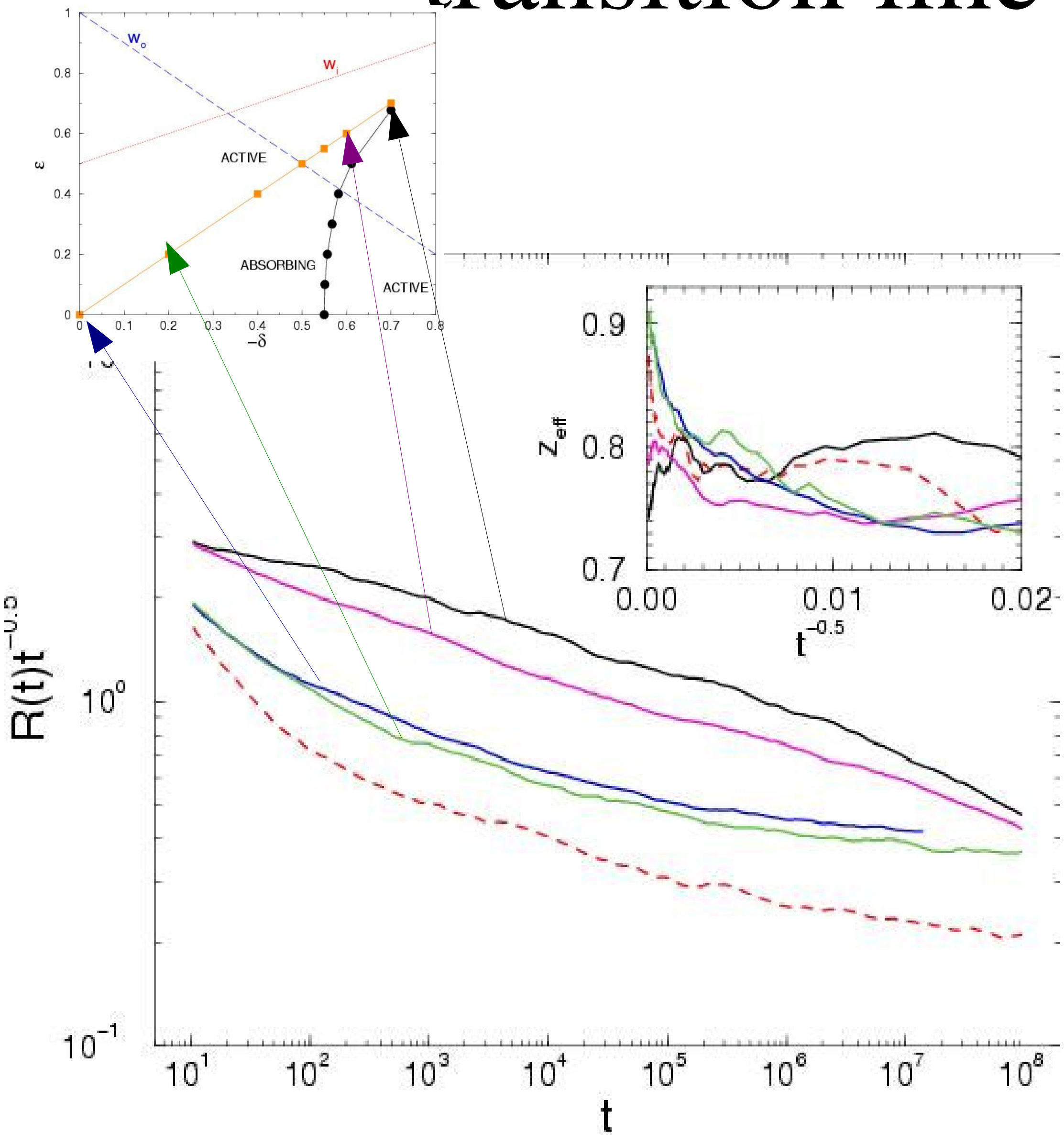


- Kink density decay from homogenous, random inintial state, L = 10^5, $t_{max} = 10^8$ MCS

$$\rho \propto t^{-\alpha}$$

- ARW + log. Corrections for weak disorder (without A-blocking)
- Increasing α (>0.5) if the annihilation is blocked as well.

Scaling behavior on the blocking transition line (clusters) II.



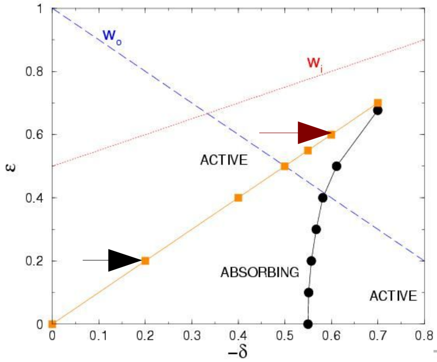
$\epsilon = 0.7, 0.6, 0.0, 0.2, 0.125$

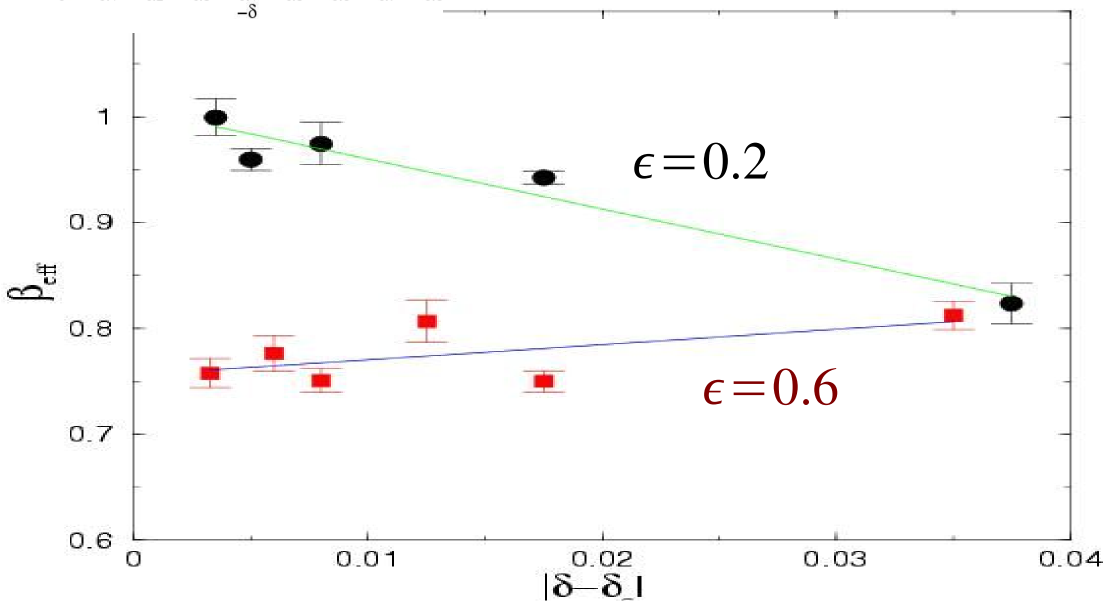
- Cluster evolution from a single kink, (odd sector), $L=10^5$, $t_{max}=10^8$ MCS

  $N \propto t^{\eta}$ $\quad$ $R \propto t^{z/2}$

- ARW + corrections (z=1) for ε < 0.5 (without annihilation blocking)
- Decreasing z if annihilation is blocked as well.
- N(∞) increases with ε, but η =0

Scaling above the blocking transition line (active fluctuating state)





- Without annihilation blocking for $\varepsilon < 0.5$: $\beta = 1$.

$$\rho \propto c_b \propto \langle l \rangle^{-1}. \tag{20}$$

For the exponential block size distribution (18) the average block size is

$$\langle l \rangle = [-\ln(p_{nw})]^{-1}, \tag{21}$$

with the no-wall probability

$$p_{nw} = 1 - p_w = 1 - (w_o + \epsilon - 1) = 2 - \epsilon - w_o. \tag{22}$$

The kink density is

$$\rho \propto -\ln(2 - \epsilon - w_o), \tag{23}$$

which for small wall probability p_w has the leading-order singularity

$$\rho \propto p_W. \tag{24}$$

TABLE II. Numerical results for the freezing transition line.

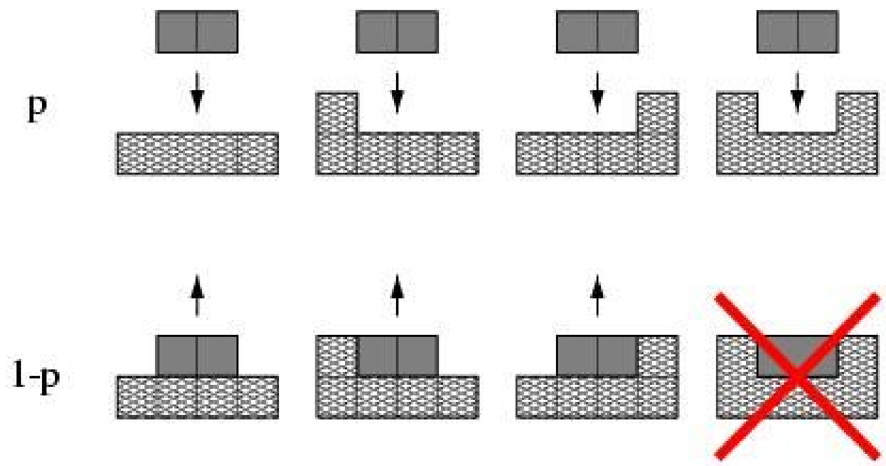
ϵ	$-\delta_c$	α	β	z	$N(\infty)$
0.2	0.2	0.47(3)	1.00(5)	0.95(5)	1.031(1)
0.4	0.4	0.47(3)		0.95(5)	1.275(5)
0.5	0.5	0.43(1)		1.0(2)	1.85(5)
0.6	0.6	0.41(1)	0.75(3)	0.76(2)	3.75(5)
0.7	0.7	0.39(1)	0.68(4)	0.72(2)	14(1)

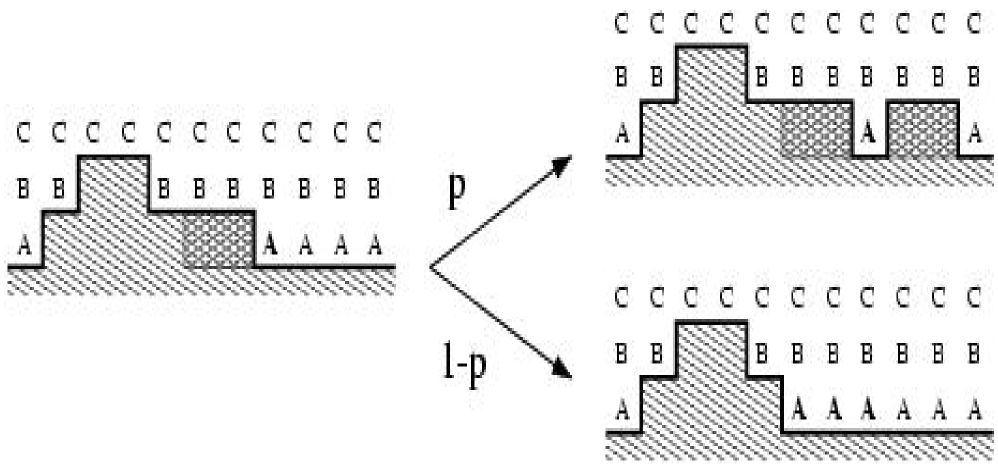
- Decreasing β if the annihilation is blocked as well.

Summary

- A PC class for weak disorder -> **Experimental observation ?** For annihilation blocking continuously changing exponents
- AA->0 stable fixed point for weak disorder (inactive phase). For strong disorder the decay slows down (continuos exponents)
- "Blocking" transition appears for strong diffusion disorder
- Published: *PRE 73 (2006) 036130*
- Acknowledgements: F. Iglói, G.M. Schütz, LCG, Clustergrid Hungary

Possible realization by a surface growth model





- Dimer growth model <--> Unidirectionally coupled parity conserving RD systems:

  A->3A B->3B C->3C ...

  2A->0 2B->0 2C->0 ...

  A->A+B B->B+C C->C+D

- Growth transition <--> absorbing phase transition at each level with parity conserving class behavior

H. Hinrichsen, G. Ódor, PRL, PRE 1999.