# The role of diffusion in branching and annihilating random walk models

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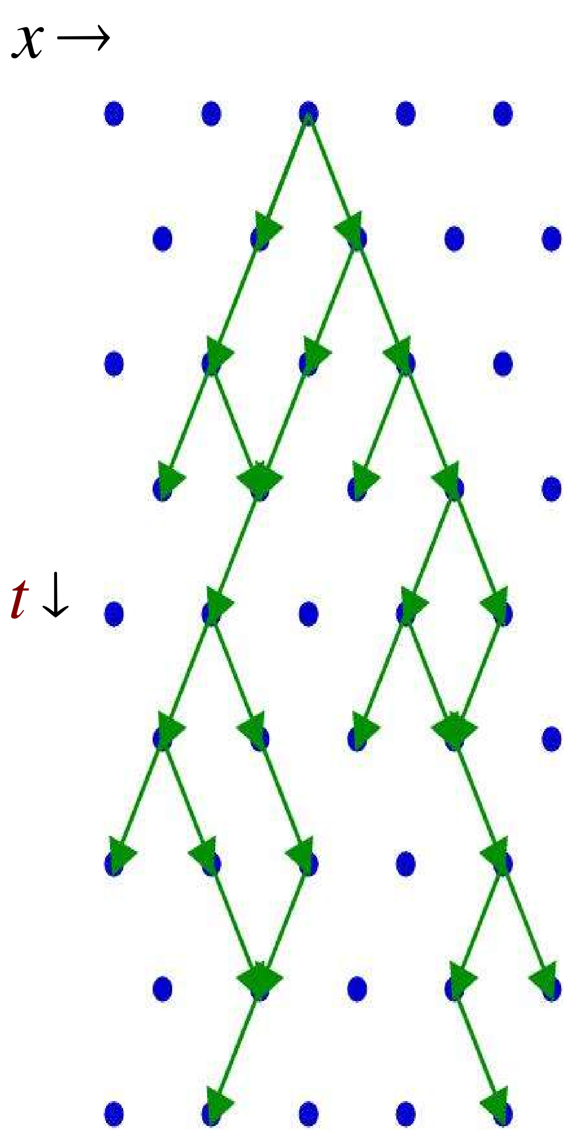
- Overview of nonequilibium universality classes of reaction-diffusion systems
- The dynamical cluster mean-field method
- Perturbatively unexpected results for simple reaction-diffusion systems with competing reactions

# Example 1: Branching and annihilating random walks (BARW) in 1+1 d




$A \overset{\sigma}{\text{->}} (n+1)A, \quad 2A \text{ -> } A, \quad A \text{ -> } 0$

<-- 1+1 d realization -->

For n=1 directed percolation (DP),
contact process, epidemic spreading
~ BARW1 ~ BARWodd (no A->0)

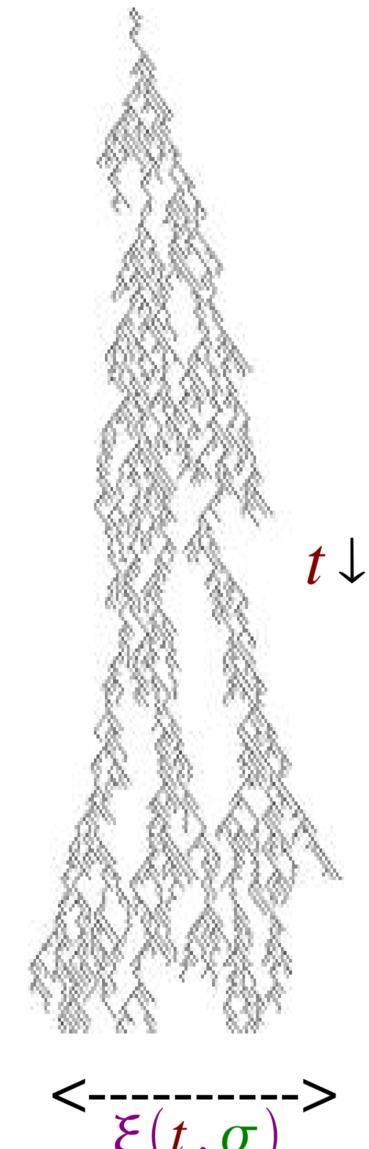
Phase transition with DP class
universal behavior:

$$\rho_\infty \propto (\sigma - \sigma_c)^\beta \qquad \rho(t) \propto t^{-\alpha}$$

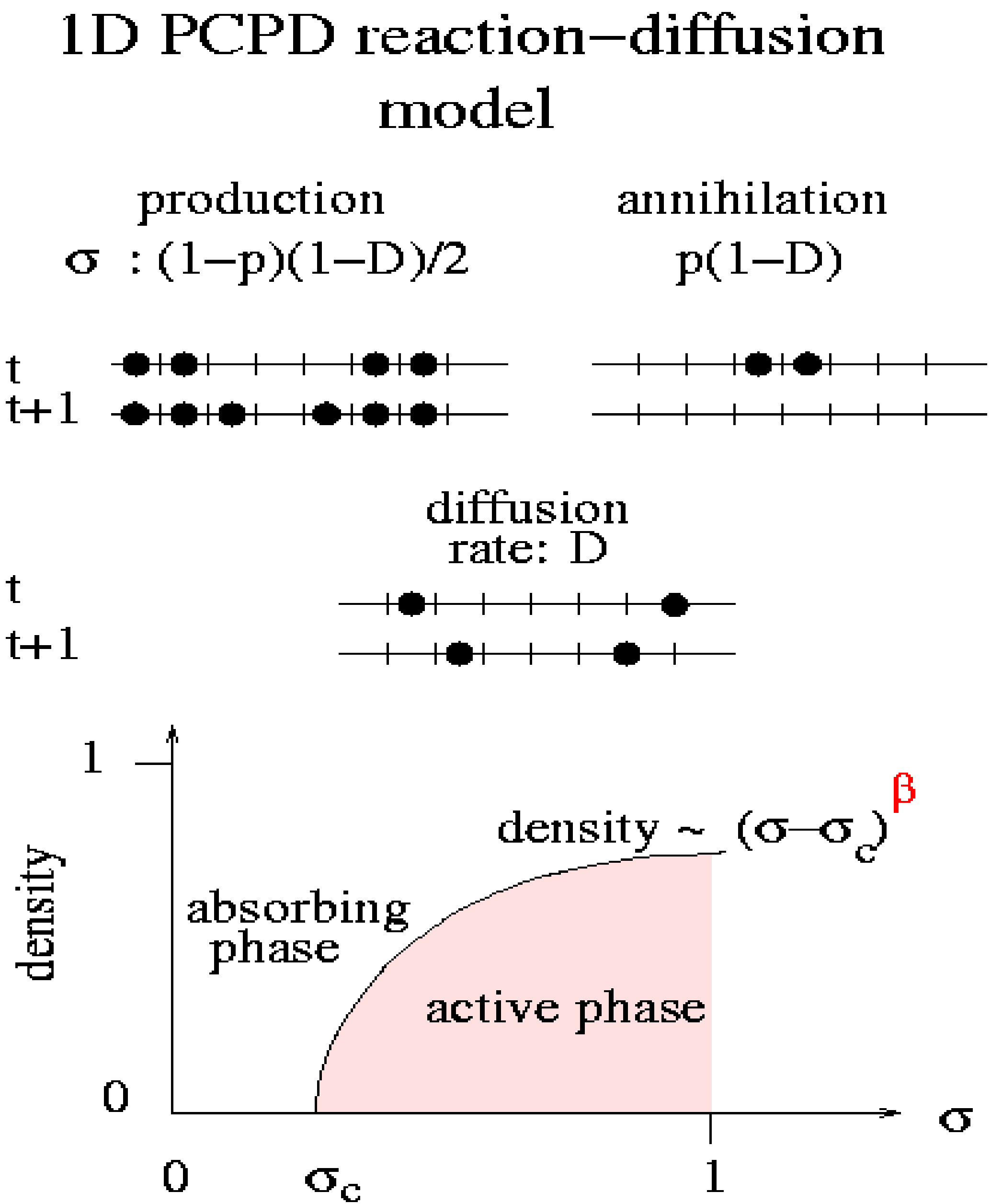
$$\xi_\infty \propto (\sigma - \sigma_c)^{-\nu} \qquad R \propto \xi \propto t^{z/2}$$

Well known exponents, field theory

Robust universality class, still lacks
experimental realization
<- sensitivity to disorder

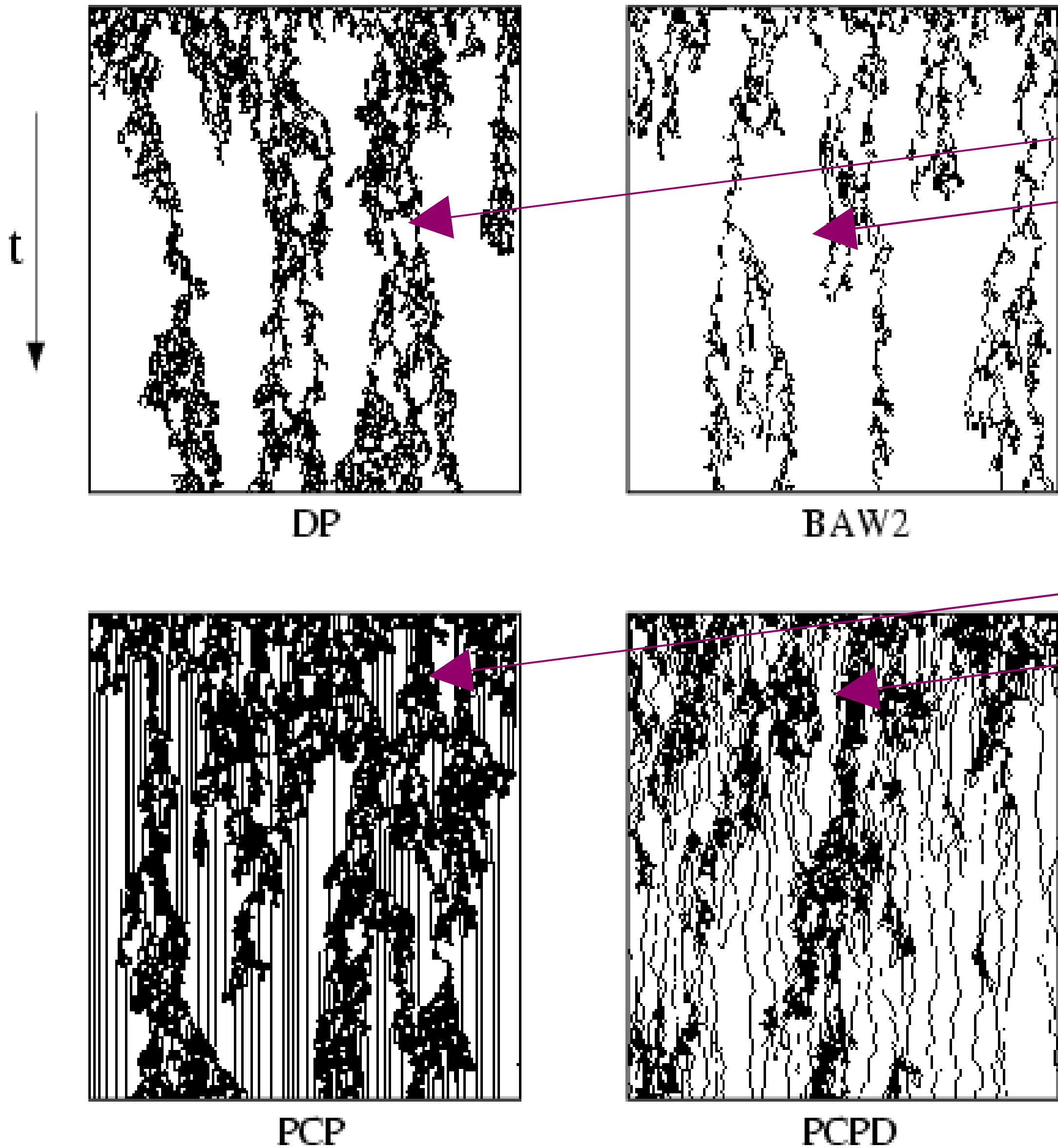


# Example 2 : Binary production (PCPD) model




- Two absorbing states **without symmetry,** one of them is diffusive. Carlon, Henkel, Schollwock (PRE 2001).
- Bosonic field theory ('97) failed to describe critical behavior. In the bosonic model diverging active phase.
- Fermionic model shows different critical behavior but field theory is too hard. Numerical methods show new exponents.
- No extra symmetries or conservation laws has been found to explain unexpceted critical behavior

# Space-time evolution of universal nonequilibrium spreading models with absorbing states in 1+1d




- **Unary** production **spreading without and with *parity conservation*:**

  **A -> (m+1)A, 2A -> 0**

- **Binary** production spreading coupled to slave modes without and with *diffusion*:

  **2A** -> (m+2)A, 2A -> 0

  Reactive and diffusive sectors, changing exponents by varying the diffusion rate: *G.Ódor, Phys. Rev. E 62 (2000) R3027.* **Two classes** ?

# Critical universality classes

- One of the primary goals of stat. phys. is to explore nonequilibriun universality classes like in equilibrium
- Due to the lack of Gibbs distribution phase transitions (PT) may occur in low dimensions
- Which factors determine the PT universility class of a model of short range interactions ? Besides the spatial dimensions, boundaries, inhomogenities:
  1) Symmetries, conservation laws like in equilibrium (BAW2 ...)
  2) Initial conditions (temporal boundary condition) (PCP ...)
  3) Topological effects in low dimensions (multi-comp systems...)
  4) Dynamically generated long range memory (coupled systems...)
  5) Mean-field classes of RD : $nA \to (n+k)A$, $mA \to (m-l)A$ --> /.
  6) For competing dynamics (diffusion) --> /.

See also: *G. Ódor, Rev. Mod. Phys. 76 (2004) 663.*

# Mean-field classes of site restricted, one-component reaction-diffusion systems

- General, reaction-diffusion systems :

Order parameter ρ

$$n A \xrightarrow{\sigma} (n+k) A \qquad m A \xrightarrow{\lambda} (m-l) A \qquad 0A \overset{D}{\Leftrightarrow} A0$$

$$\rho(t) \propto t^{-\alpha}$$
$$\rho(\infty) \propto \epsilon^{\beta}$$

$$\partial \rho / \partial t = a k \sigma \rho^{n} (1-\rho)^{k} - a l \lambda \rho^{m}$$

- $n = m$ : $\beta = 1, \alpha = 1/n$ $\quad \sigma_c = l/(k+l)$
- $n < m$ : $\beta = 1/(m-n), \alpha = 1/(m-1)$ $\quad \sigma_c = 0$
- $n > m$ : First order transition

***n* and *m* determine the (site) mean-field class!**
**Diffusion does not play a role.**

*G. Ódor: PRE 67, 056114 (2003)*

# The dynamical, *N*-cluster mean-field method (GMF)

- Master equation for *n*-point configuration probabilities of $s_i$

$$\frac{\partial P_n(\{s_i\})}{\partial t} = f(P_n(\{s_i\})), \qquad (1)$$

- Bayesian extension process ($n>N$ correlations are neglected)

$$P_n(s_1, \ldots, s_n) = \frac{\prod_{j=0}^{j=n-N} P_N(s_{1+j}, \ldots, s_{N+j})}{\prod_{j=1}^{j=n-N} P_{N-1}(s_{1+j}, \ldots, s_{N-1+j})}. \qquad (2)$$

- Reduction of parameters due to svmmetries. conservations

$$P_n(s_1, \ldots, s_n) = \sum_{s_{n+1}} P_{n+1}(s_1, \ldots, s_n, s_{n+1}),$$

$$P_n(s_1, \ldots, s_n) = \sum_{s_0} P_{n+1}(s_0, s_1, \ldots, s_n).$$

- If we apply GMF for one-dimensional, site restricted lattice version of BARWe. For *N=10* we have *528* independent variables

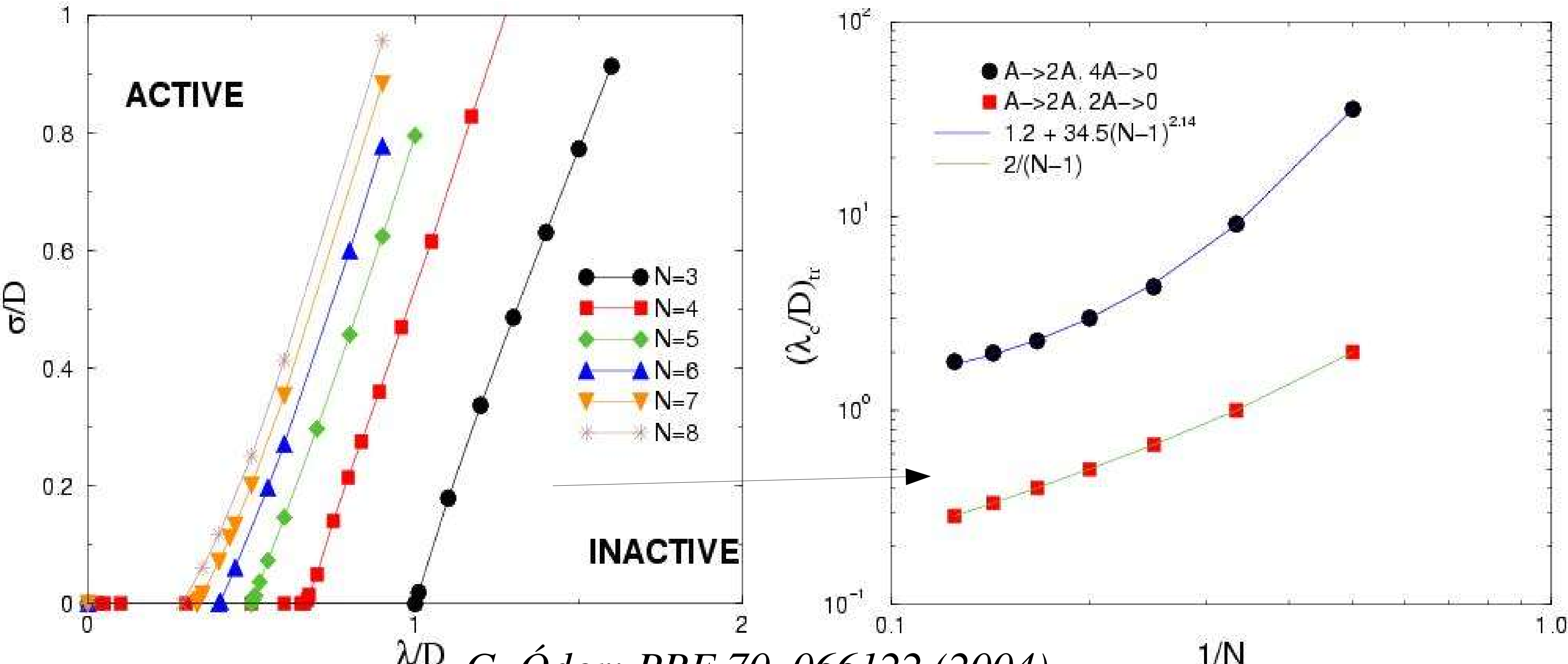
# Cluster mean-field for BARW($m>n$)

$$(site\,MF: m>n: \beta=1/(m-n), \alpha=1/(m-1), \sigma_c=0)$$

1) *A -> 2A, 2A -> 0 :*

Exact relation: $(\lambda_c/D)_{tr} = 2/(N-1)$ found : $(\lambda_c/D)_{tr}$ -> 0 as $N$ -> *inf.*

2) *A -> 2A, 4A -> 0*: $(\lambda_c/D)_{tr}$ -> ~ 1.2 as $N$ -> *inf.*



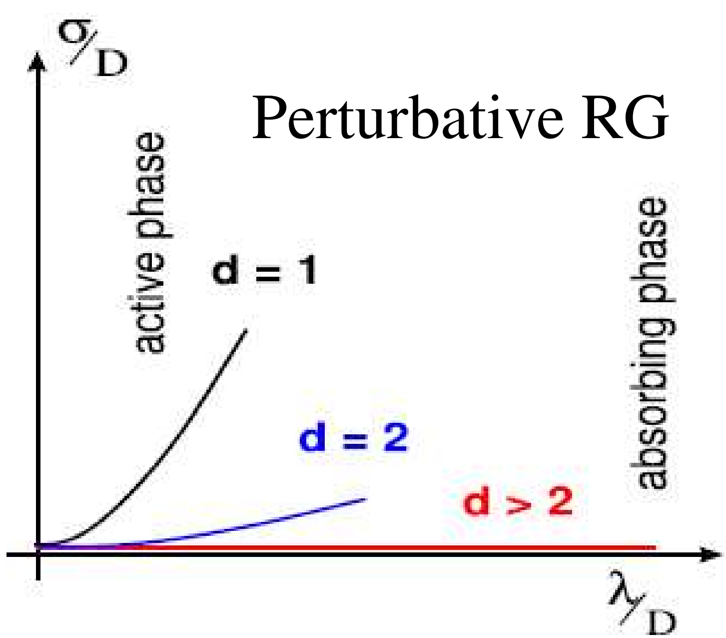

*G. Ódor: PRE 70, 066122 (2004)*

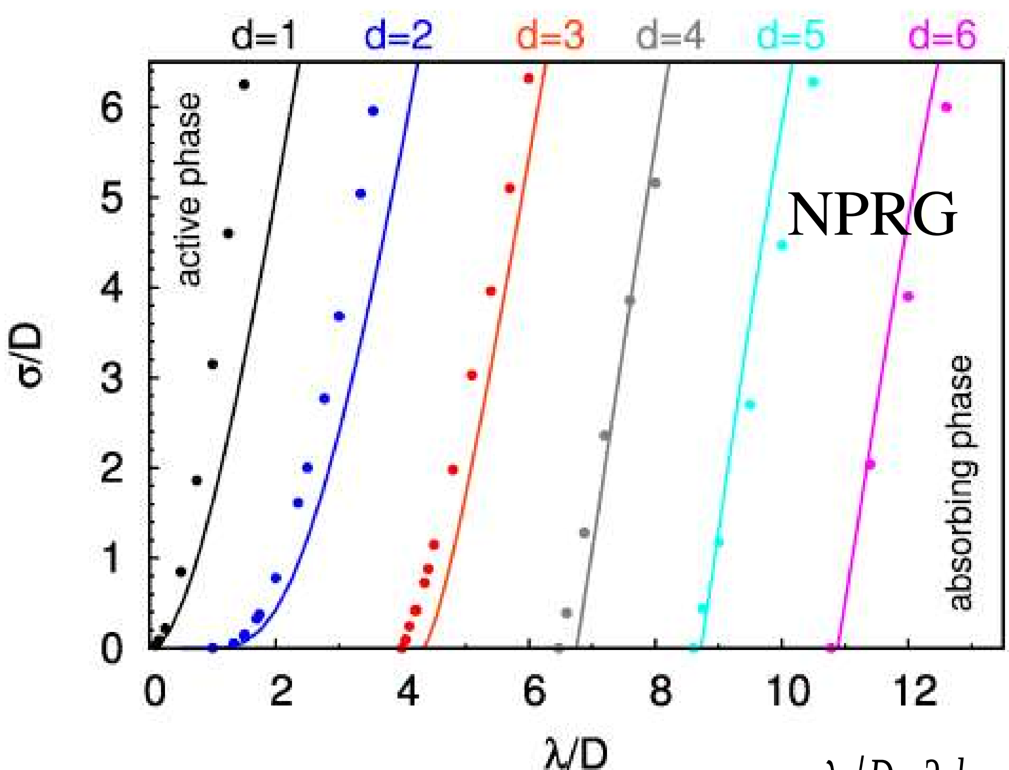
# ERG field theoretical reminder (B. Delamotte et al.)

General reactions: $A \xrightarrow{\sigma} mA\,,\; kA \xrightarrow{\lambda} 0$ (Cardy & Tauber)

$$\mathcal{S}[\phi,\hat{\phi}] = \int d^d x\, dt \Big\{ \hat{\phi}(x,t)\left(\partial_t - D\nabla^2\right)\phi(x,t) - \lambda_k \left(1-\hat{\phi}(x,t)^k\right)\phi(x,t)^k + \sigma_m\left(1-\hat{\phi}(x,t)^m\right)\hat{\phi}(x,t)\,\phi(x,t) \Big\}$$

$A \rightarrow 2A\,,\; 2A \rightarrow 0$



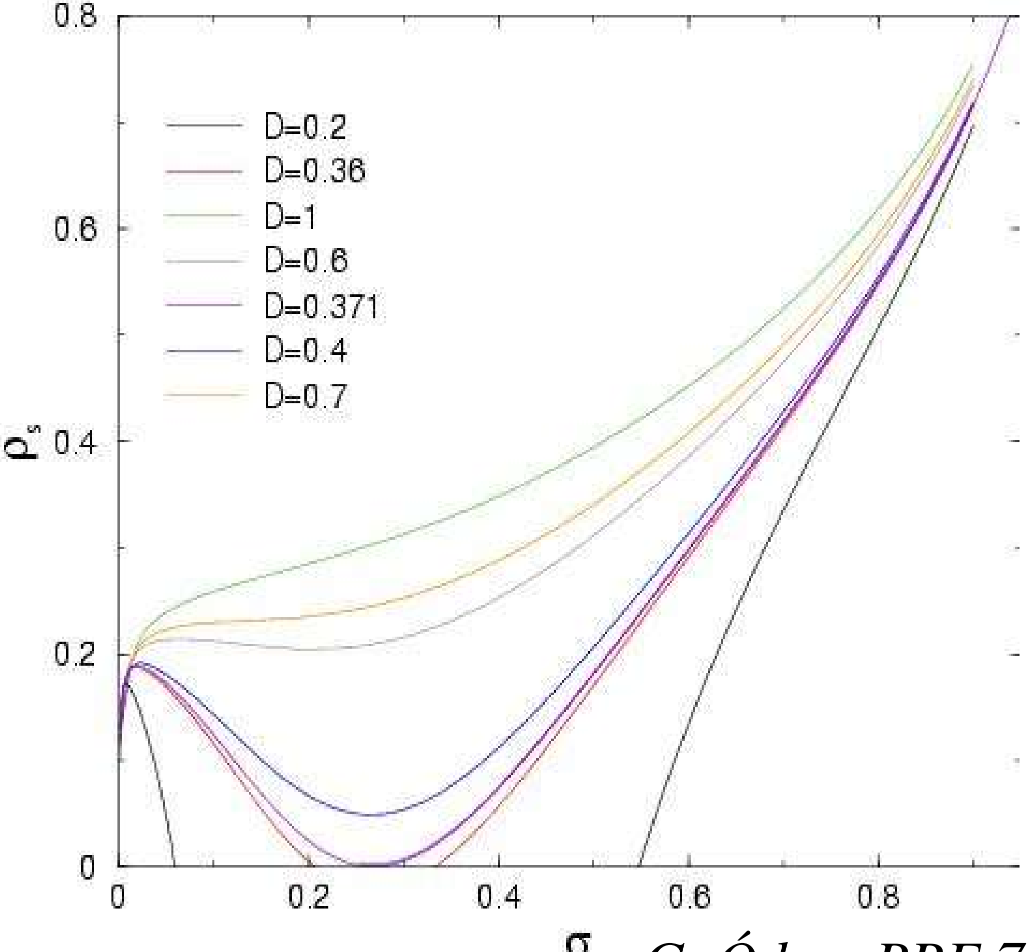




$\lambda_c / D = 2d$

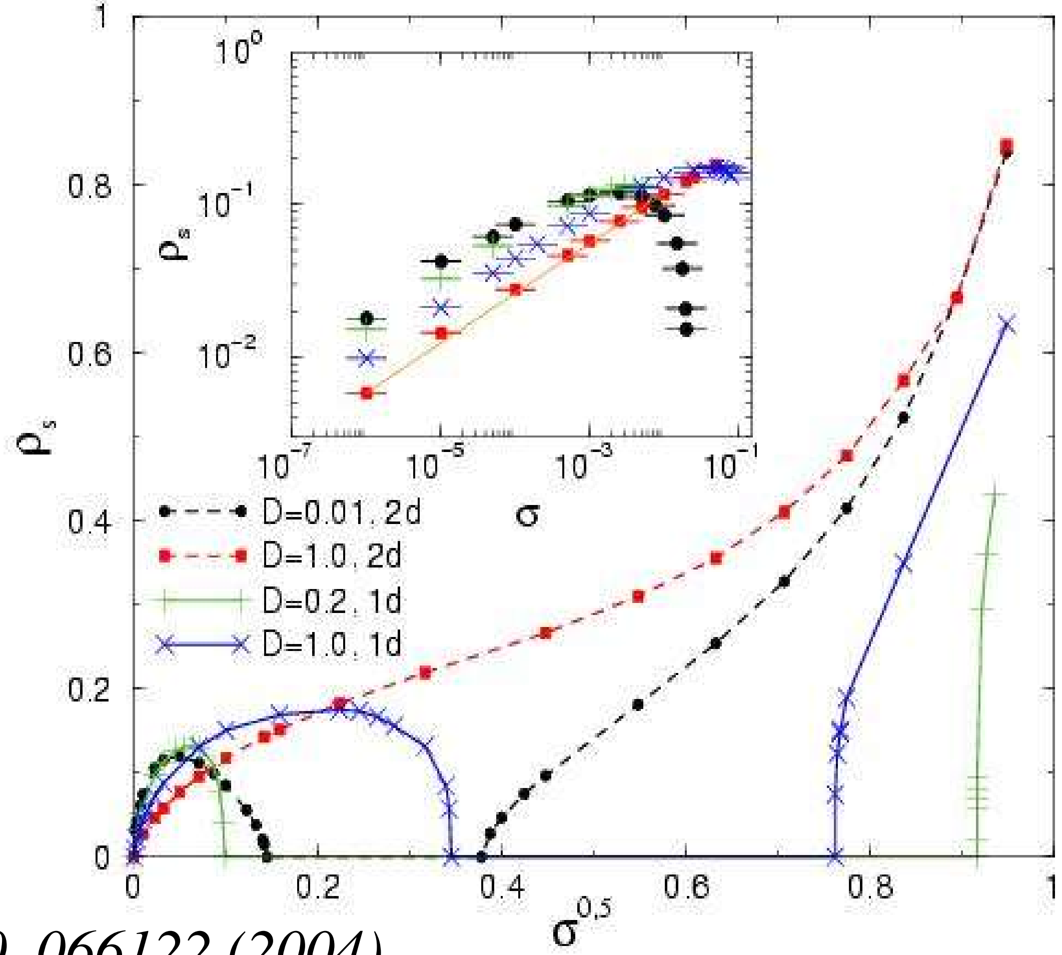
(Canet, Chaté, Delamotte, *PRL 2004,* NPRG)

# *A->2A, 4A->0* : Reentrant phase diagram

*N*-cluster approximation




Simulations:
*2d: $L^2=4000^2$, 1d: $L= 10^5$*

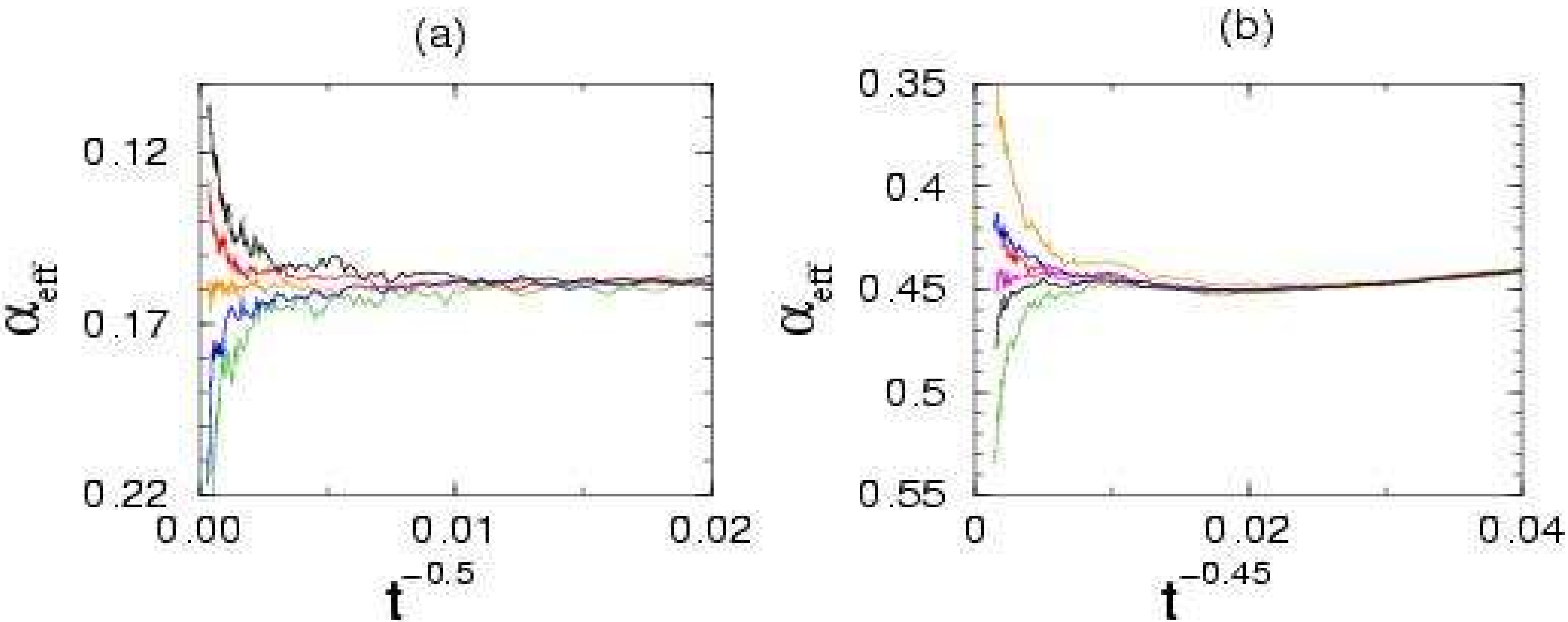



*G. Ódor: PRE 70, 066122 (2004)*

# Critical point decay at $\sigma_c > 0$ in (a): one and (b): two dimensions

DP class value in 1d: $\alpha$= *0.159464(6)* (I. Jensen)

2d: $\alpha$= *0.4505(10)* (Voigt and Ziff)



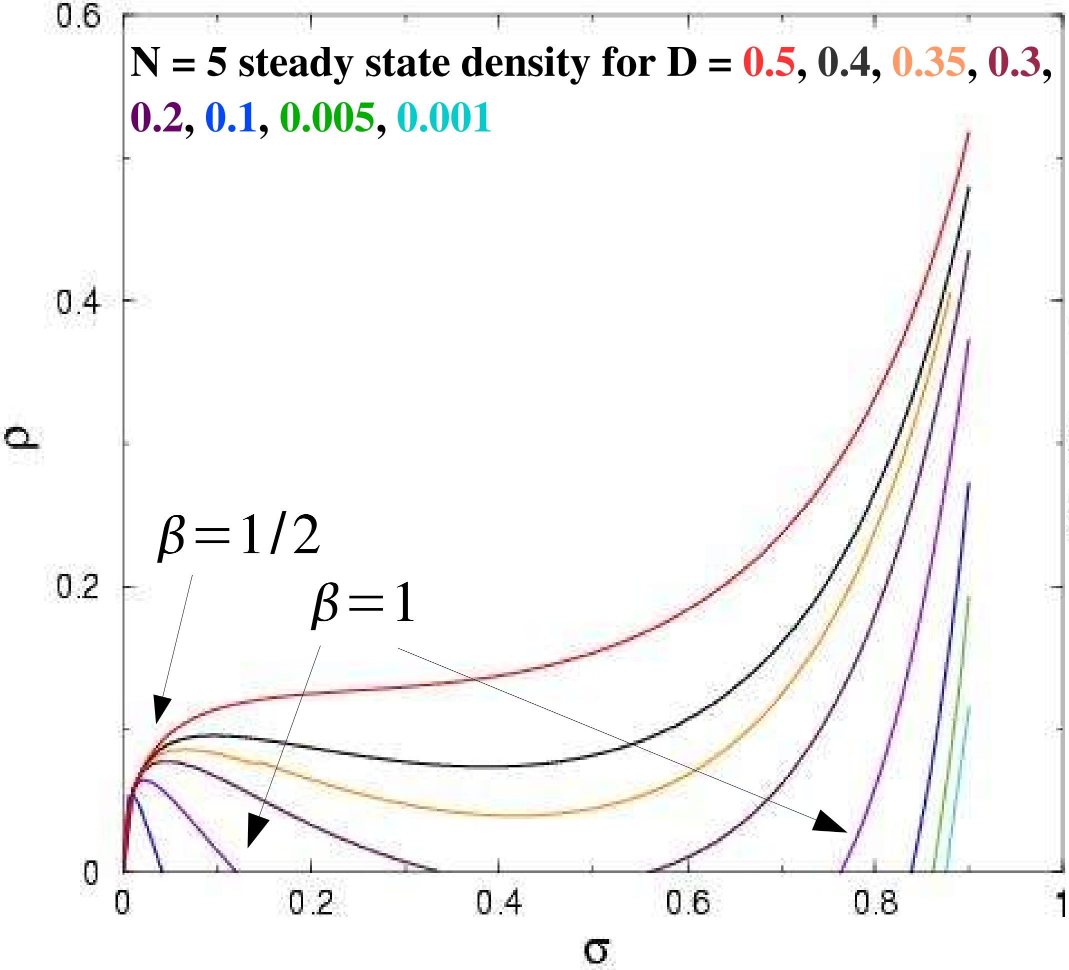

**For low diffusions (and high d) the : *A->2A->3A->4A->0* process becomes relevant ! Unexpected by perturbative RG.**

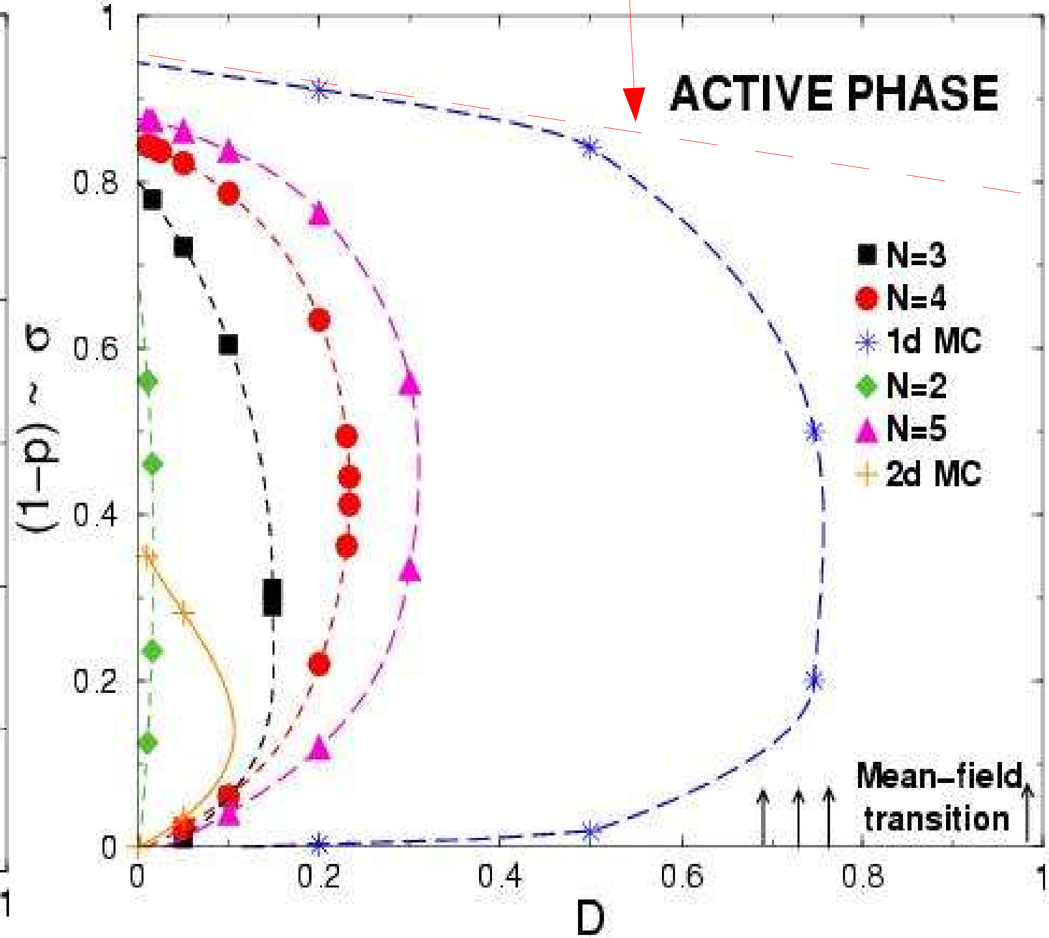
# Cluster approximations for: 2A -> 3A, 2A -> 0 model

Perturbative RG expectations

Steady state density for $N >= 2$ (diffusion dependence).

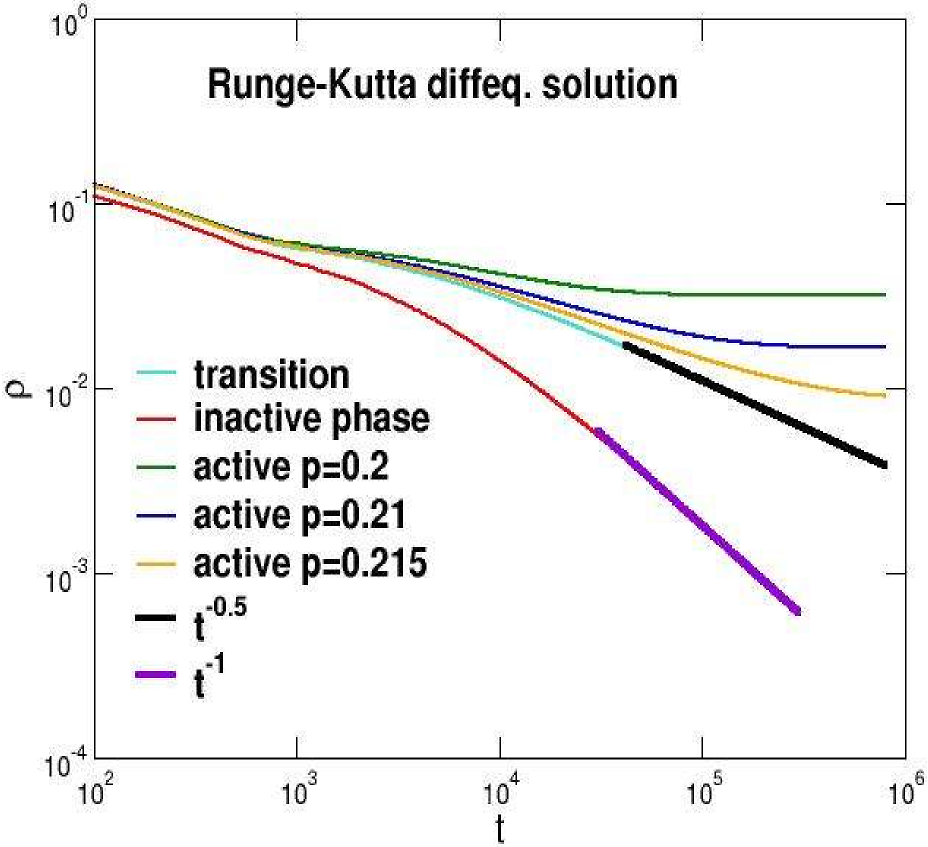
Unexpected phase transitions for $\sigma_c > 0$, with $\beta$=*1* (*for n < m site MF*: $\sigma_c$ = *0*)



# Cluster approximations for: *2A -> 3A, 4A -> 0* model

**Density decay solution for *N = 3* at *D=0.05* near $\sigma_c$ > 0 critical point**
**with exponent $\alpha = 0.5$ ( *2A -> 3A, 2A -> 0* (PCPD) behavior )**

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Simulations in 1 and 2 dimension support *N*-cluster results:
*G. Ódor, PRE 69, 036112 (2004)*
*G. Ódor: PRE 70, 026119 (2004)*

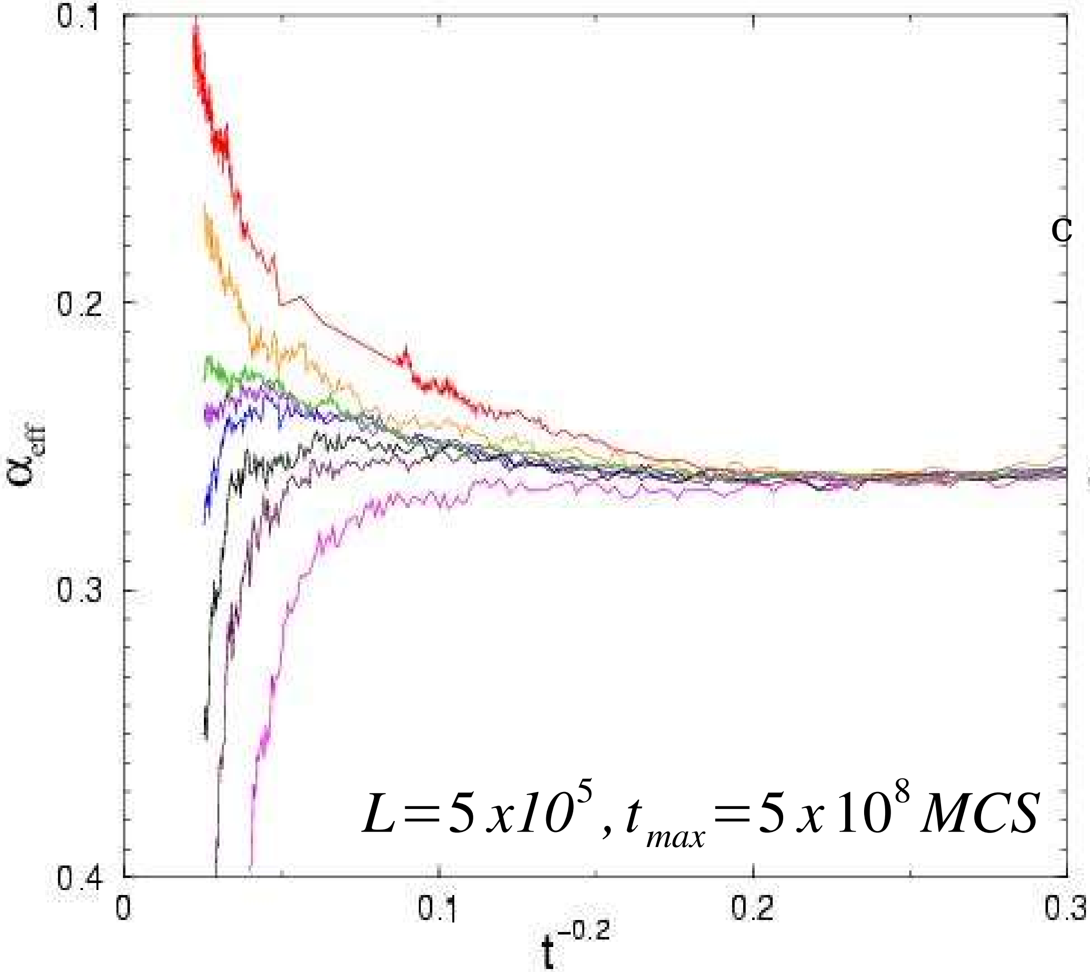
Unexpected by perturbative RG

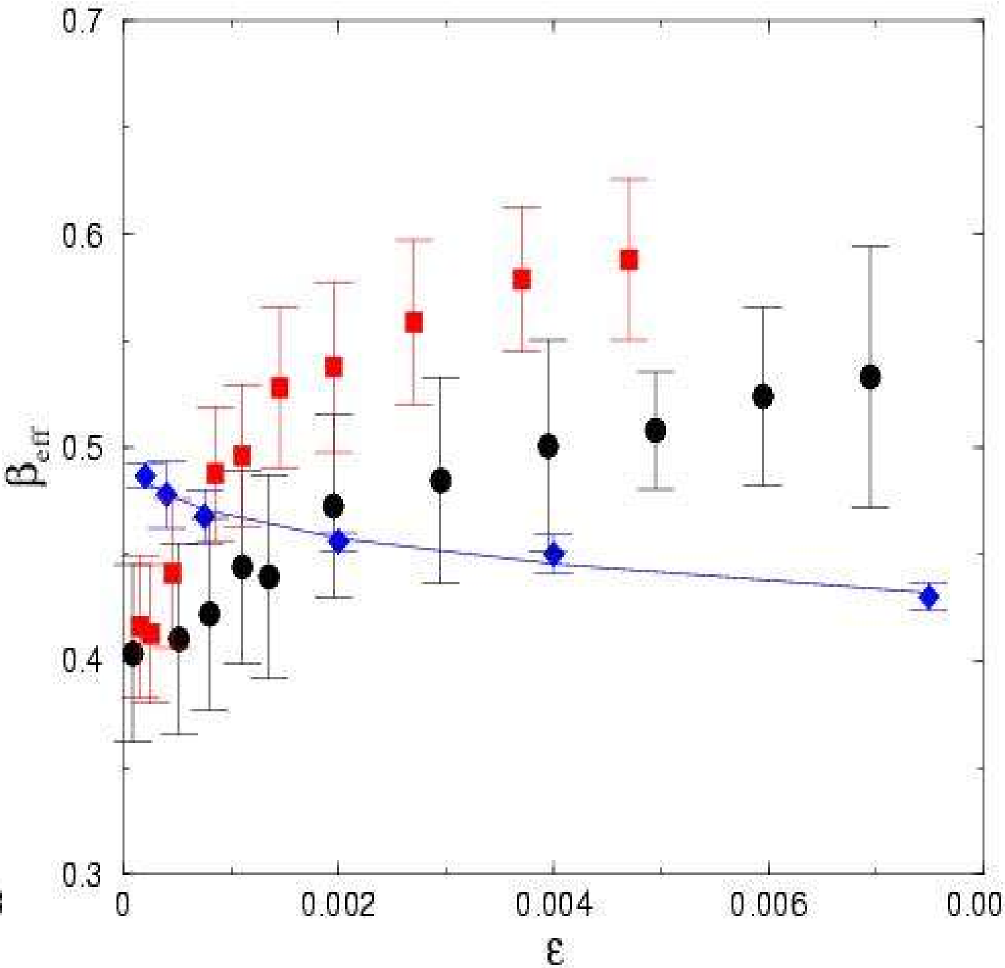
**For low diffusions the : *2A->3A->4A->0* process becomes relevant !**

# Simulation results for the *2A -> 3A, 4A -> 0* model in 1 dimension

Density decay local exponents with : $\alpha$= *0.21(2)* (~ PCPD), at $D = 0.5$, $\sigma_c = 0.42075$.

Steady state local exponents near $\sigma_c = 0.42$ for $D = 0.5$, *0.2*, with $\beta$= 0.40(2) (~PCPD).
For $D = 0.9$: only mean-field with $\beta$=1/2.
*2A -> 0* process becomes irrelavant !



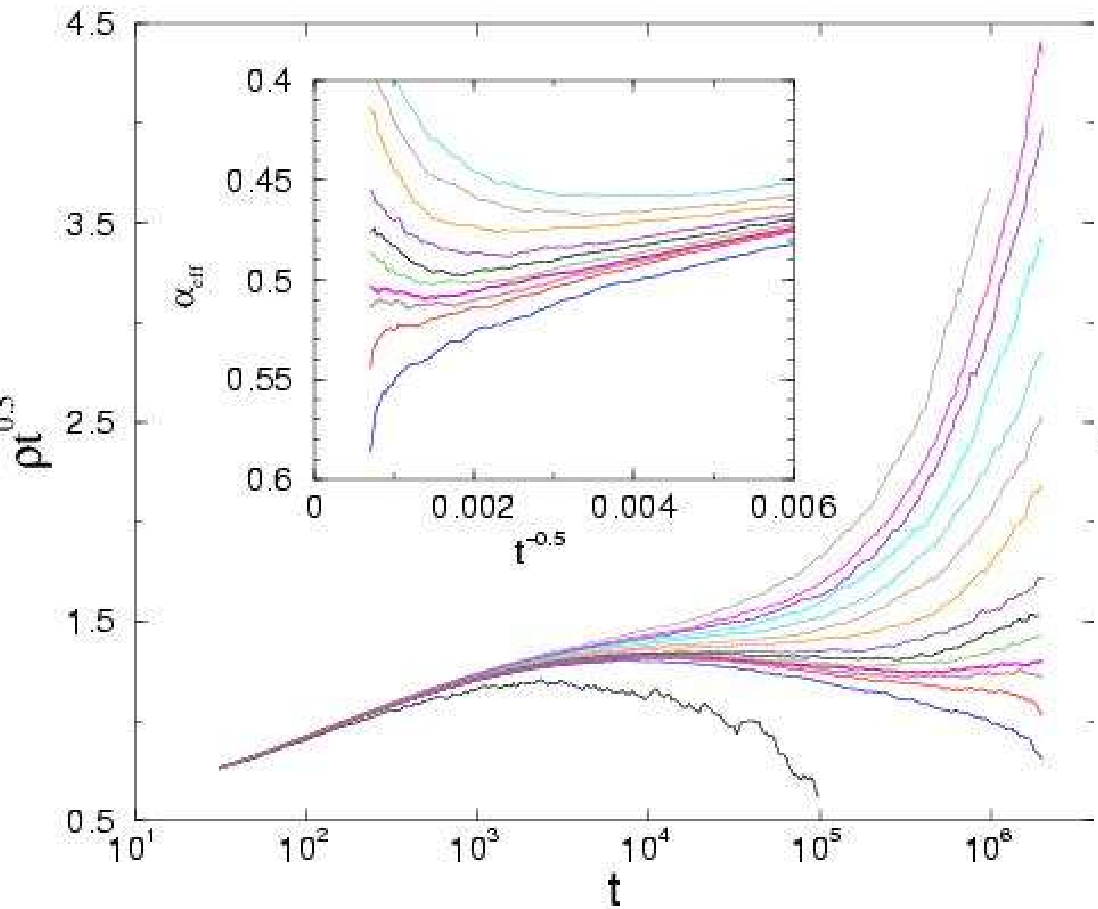



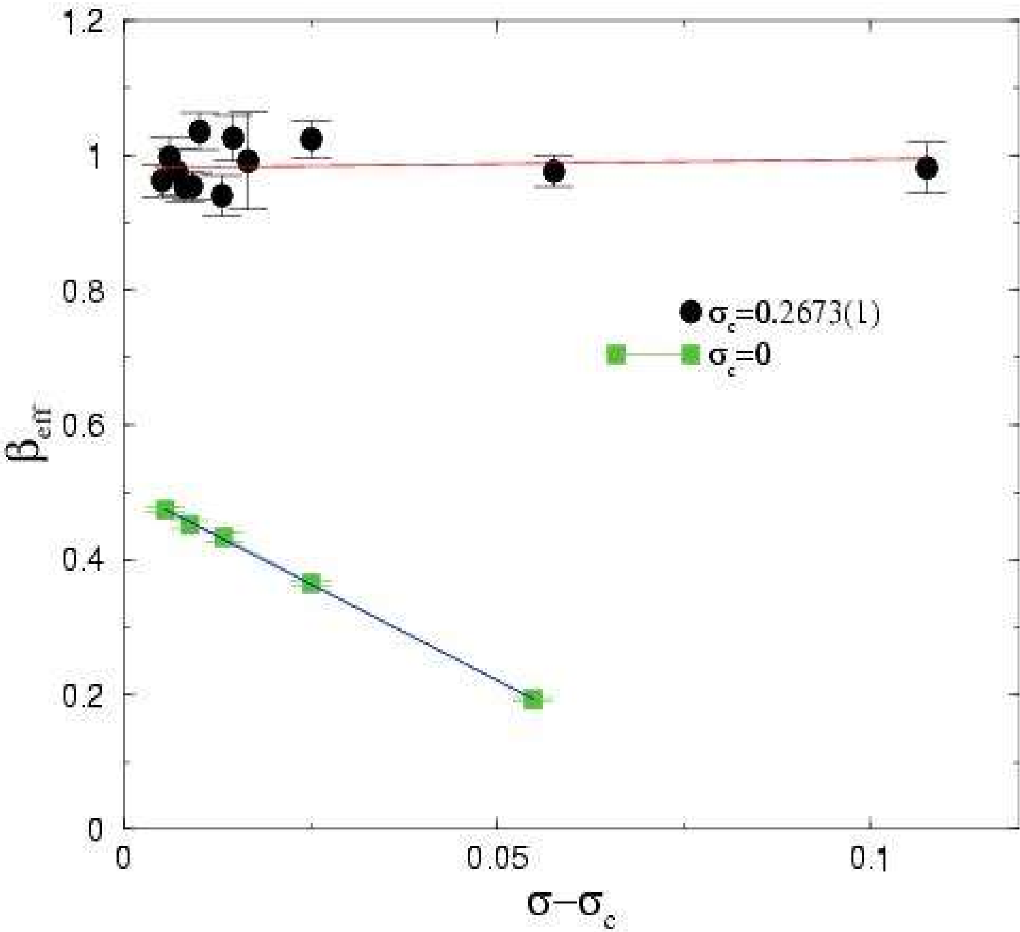
# Simulation results for the *2A -> 3A, 4A -> 0* model in 2 dimensions

Density decay local exponents with :
$\alpha = 0.50(1)$ (MF-PCPD) at $\sigma_c = 0.26715$

$L = 7000, t_{max} = 2\,x\,10^6\,MCS, D = 0.05$

Steady state local exponents at $D = 0.05$
with $\beta = 0.98(2)$ (MF-PCPD).
For $D = 0.9$: only mean-field with $\beta = 1/2$.
*2A -> 0* process becomes irrelavant !



# Summary

- In *nA->(n+k), mA->(m-l)A* type reaction-diffusion systems the mean-field universality class is determined by the number of reacting particles (*n*,*m*)
- Diffusion may become relevant in case of competing reactions by changing the phase diagram and introducing nontrivial fixed points
- Diffusion dependence has been found in similar systems:
  - *R. Dickman*: A -> 2A, 3A -> 0, *PRA42, 6985 (1990)*
  - *M. Paessens, G. Schütz :* bosonic PCPD, *JPA37, 4709 (2004)*
  - *N. Menyhárd, G. Ódor :* NEKIM-A, *PRE68, 056106 (2003)*
- Perturbative RG results may not hold for low Diffusion and high dim !
- References : *G. Ódor: PRE 67, (2003) 056114*
  *G. Ódor: PRE 67, (2003) 016111*
  *G. Ódor: PRE 69, (2004) 036112*
  *G. Ódor: Rev. Mod. Phys. 76, (2004) 663 (cond-mat/0205644)*
  *G. Ódor: PRE 70, (2004) 026119*
  *G. Ódor: PRE 70, (2004) 066122*