

Relaxation of initial conditions in systems with infinitely many absorbing states

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We have investigated the effect of the initial condition on the spreading exponents of the one-dimensional pair contact process (PCP) and threshold transfer process. The nonorder field was found to exhibit critical fluctuations, relaxing to its natural value with the same power law as the order parameter field. We argue that this slow relaxation, which was not taken into account in earlier studies of these models, is responsible for the continuously changing survival probability exponent. High-precision numerical simulations show evidence of a (slight) dependence of the location of the transition point on the initial concentration, in the case of PCP. The damage spreading point and the spreading exponents coincide with those of the ordinary critical point in both cases. [S1063-651X(98)01112-X]

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I. INTRODUCTION

Recently the question of whether one can construct initial states that affect the *entire* temporal evolution of critical non-equilibrium systems was addressed. This is the case of systems that display a phase transition between an active state and a phase with infinitely many absorbing states. Nonuniversality of dynamic properties, associated with the initial configuration dependence of the survival probability of clusters started from a single active site, has been reported [1–3].

Similar changes in the critical spreading behavior have been observed by Grassberger, Chaté, and Rousseau [4] in a model where long-time memory effects are explicitly introduced. In the system studied by these authors, the susceptibility to the spreading of an active agent changes after the first encounter, remaining constant afterwards. Despite the observed nonuniversal dynamical critical behavior, susceptibility in the first encounter does not affect the critical point location. Grassberger, Chaté, and Rousseau argue that these results apply to models with multiple absorbing states where an effective memory-dependent susceptibility is present. However, different behavior is predicted in the case of slowly decaying memory effects, in which case both the value of the critical point and the exponents are expected to be modified.

A dependence on the initial configuration has also been found [5] in the case of long-range spatial correlations, in which case the dynamical critical exponents change continuously as a function of initial correlation length. The critical relaxation from an initial homogeneous state in systems with multiple absorbing states was recently examined from a field theoretical (Langevin equations) approach [6]. The evolution equation for the order-parameter density was found to include a memory term that is not present in the simpler case of directed percolation (DP).

In the present study we have investigated memory effects in two $1d$ models with multiple absorbing states, the pair contact process (PCP) [7,1] and threshold transfer process (TTP) [2] models. In both models there is a nonorder-parameter field, dynamically coupled to the order-parameter

field, that gives rise to an effective susceptibility for spreading. We show that the nonordering field relaxes to its steady state value by the same power-law time dependence as the order-parameter field and that this is characterized by the natural, long-time behavior exponent of the density decay of a DP process. This is clear evidence of slowly decaying memory. This power-law boundary condition (in time) is similar to that of the long-range power-law boundary condition (in space) of [5] and we can see the emergence of continuously changing dynamical critical exponents. The small shift of the critical point as a function of initial conditions, shown by high-precision simulations for the PCP model, is in agreement with the above arguments.

Damage spreading (DS) simulations invented in biology [8] and later in physics [9] are useful to show the stability of the systems with respect to small perturbations. The spreading behavior has been shown to be sensitive to the dynamics leading to the same steady states. An “objective” definition of DS has been proposed [10] according to which the phase diagrams of the steady states of nonequilibrium models can be divided to sectors in which all, none or parts of the physically possible dynamical rules generate stable damages. The phase transitions between the phases can be continuous and usually belong to the DP universality class [11]. However, if the damage variables possess conservation and the DS absorbing states exhibit symmetries—which usually happens when the DS transition point coincides with the ordinary critical point the DS transition can belong to a different universality class [12,13]. The DS transition cannot be in the passive phase of the replicas, but if it occurs in the active phase, the fluctuating replicas at the DS absorption point exclude the non-DP DS behavior [13]. We have investigated the DS properties of the PCP and TTP models; the DS transition is shown to coincide with the ordinary critical point, and the nonuniversal spreading exponents have been inherited as well.

In Sec. II, we give a brief introduction to the PCP and TTP models. Time-dependent and critical relaxation studies are described in Sec. III, whereas Sec. IV is devoted to dam-

age spreading simulations. Some comments and conclusions are presented in Sec. V.

II. PCP AND TTP MODELS

Both of these models have a single control parameter p and qualitatively similar phase diagrams, displaying an “active state” ($p < p_c$ in the case of PCP, $p > p_c$ for TTP) and (infinitely many) absorbing phases ($p > p_c$ in the case of PCP, $p < p_c$ for TTP).

In the TTP model, each site may be vacant, single, or doubly (active) occupied, and this can be described by a three-state variable $\sigma_i = 0, 1, 2$. In each time step, a site is chosen at random in the infected region. In the absence of active sites, the dynamics is indeed trivial: if $\sigma_i(t) = 0$ (or 1), then $\sigma_i(t+1) = 1(0)$ with probability p (or $1-p$). The system relaxes exponentially to a steady state where a fraction p of sites have $\sigma_i = 1$ and the others are vacant. If $\sigma_i(t) = 2$, then $\sigma_i(t+1) = 0, \sigma_{i-1}(t+1) = \sigma_{i-1}(t) + 1, \sigma_{i+1}(t+1) = \sigma_{i+1}(t) + 1$ if $\sigma_{i+1}(t)$ and $\sigma_{i-1}(t)$ are both < 2 and $\sigma_i(t+1) = 1$ if only one of the nearest neighbors of site i ($j = i-1$ or $i+1$) has $\sigma_j(t) < 2$, in which case $\sigma_j(t+1) = \sigma_j(t) + 1$. As can be easily seen, the number of active sites either decreases or remains the same in all processes other than $(1, 2, 1) \rightarrow (2, 0, 2)$; the frequency of these processes depends on the concentration of 1’s, which is controlled by the parameter p . Any configuration consisting of only 0’s or 1’s is absorbing in what concerns the active sites. The absorbing states in this model are fluctuating—in the respective sector of phase space, ergodicity is not broken.

As we show below, the dynamics of 1’s is, however, strongly affected by the presence of active sites. At the critical point, the concentration of 1’s relaxes to its steady state value (equal to p_c) by a power law.

The PCP is a two-state variable model with multiple absorbing states, each one of them completely frozen in time, contrary to what happens in the TTP. In the PCP, nearest-neighbor pairs of particles (dimers or active sites) annihilate each other with probability p or create, with probability $1-p$, a particle at one of the adjacent (vacant) sites to the dimer. Dimers cannot be generated spontaneously and therefore play the role of the 2’s in the TTP. There is a natural configuration to which the system at criticality evolves after all activity has died out; the relationship between the natural particle density and p_c is not a simple one unlike the TTP case. As shown below, the relaxation to this natural state is a slow process—a power law in time is also found in this case and slowly decaying memory effects arise as a result of the coupling between the local density of dimers and the local density of isolated occupied sites.

III. TIME-DEPENDENT SIMULATION RESULTS

Time-dependent simulations [14] have become an effective tool to explore dynamical critical exponents of systems at nonequilibrium phase transition points. The simulations are started from a single active seed embedded in a sea of inactive sites and followed up to some t_{MAX} time such that the cluster size cannot exceed the system size L . The quantities usually investigated are the mean number of active sites $N(t)$ (pairs in the case of the PCP and 2’s in the case of the

TTP model) averaged over all trial samples, the survival probability $P(t)$ of the clusters and the mean spreading size $R(t)$ of the surviving clusters. At the critical point and for asymptotically long times these quantities exhibit power-law behavior like

$$N(t) \propto t^\eta, \quad (1)$$

$$P(t) \propto t^{-\delta}, \quad (2)$$

$$R(t) \propto t^{z/2}, \quad (3)$$

which define the exponents η , δ , and z , respectively. The cluster size exponent z characterizing the linear scale is related to the anisotropy exponent of the system $Z = \nu_{\parallel} / \nu_{\perp}$ by $Z = 2/z$. The order-parameter density inside the surviving clusters can be expressed in terms of these exponents as

$$\rho(t) \propto t^{\eta + \delta - dz/2} \quad (4)$$

and is expected to show the same long-time decay as from an arbitrary bulk configuration with $\rho(0) \neq 0$, i.e.,

$$\rho(t) \propto t^{-\beta/\nu_{\parallel}} \quad (5)$$

where β is the steady state order-parameter exponent.

Exponents η and δ are found to depend on the initial concentration of particles $\rho_1(0)$, and are related to static exponents β and ν_{\parallel} by the hyperscaling relation [2]

$$2\eta + 2(\delta + \beta/\nu_{\parallel}) = dz. \quad (6)$$

Seed growing simulations for the PCP and TTP models have been carried out up to $t_{\text{MAX}} = 8000-16000$ time steps (MCS) for 2×10^5 trial runs. We measured the order parameter density $\rho_2(t)$ as well as the relaxation of the nonorder field density $\rho_1(t)$ towards the natural values $\rho_1^{\text{nat}} \equiv \rho_1(\infty)$ of the models. In case of the PCP we used $\rho_1^{\text{nat}} = 0.242(1)$ [15] while for the TTP model $\rho_1^{\text{nat}} = p_c = 0.6894(3)$ [2]. The densities were measured inside the “infected” regions of surviving clusters only. To estimate the critical exponents and the transition points together, we determined the local slopes of the scaling variables. For example, in the case of the order parameter density we computed

$$-\alpha(t) \equiv \frac{\ln[\rho_2(t)/\rho_2(t/m)]}{\ln(m)} \quad (7)$$

with $m = 8$. When $p = p_c$, one should see a straight line as $1/t \rightarrow 0$. The off-critical curves should possess curvature: curves corresponding to $p > p_c$ should veer upward, curves with $p < p_c$ should veer downward.

Figures 1 and 2 show the local slopes of $\Delta\rho_1(t) \equiv \rho_1(t) - \rho_1^{\text{nat}}$ for the PCP in case of $\rho_1(0) = 0$ and $\rho_1(0) = 0.432$ respectively. For the order parameter density we obtained the same results within numerical accuracy. As one can read off, the particle density exhibits long-time power-law behavior with DP exponent, but the critical point is slightly lower in the case of $\rho_1(0) = 0$ than in case of $\rho_1(0) = 0.432$. This, however, agrees with the slow-relaxing susceptibility picture, because $\rho_1(0) = 0$ is smaller than the natural value $\rho_1^{\text{nat}} = 0.242(1)$ and increases very slowly in the bulk, there-

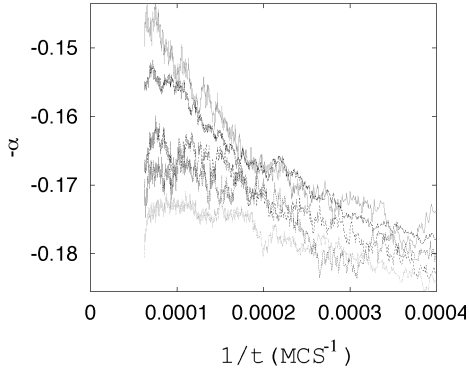


FIG. 1. Local slopes of $\Delta\rho_1(t)$ in PCP simulations at $\rho_1(0)=0$ for $p=0.077, 0.07695, 0.0769, 0.07683, 0.07675$ (from bottom to top curves). Scaling can be observed for $p=0.07685(5)$, with the exponent $\alpha(t\rightarrow\infty)\approx 0.16$.

fore a smaller annihilation probability is enough to drive the system to the absorbing state. This dependence can also be the result of the lattice version of the PCP model, where, in contrast to the field theory where there is always some finite density of fields, the isolated 1's are frozen. Both densities exhibit the same exponents in good agreement with $\beta/\nu_{\parallel}=0.1596(4)$ of the $1+1$ d DP class. The survival probability exponent measured in our high precision simulation is in agreement with the value of [1] within numerical accuracy.

In the critical point estimates we could see a tiny difference depending on whether we measured the survival probability or the density in the same simulation. Namely, the p_c appears to be nearer to the results of [1,17] if we estimated it from the $P(t)$ data ($p_c=0.07704$ instead of $p_c=0.07685$ in the case of $\rho_1(0)=0$ and $p_c=0.07714$ instead of $p_c=0.0770$ in the case of $\rho_1(0)=0.432$). This small offset can be understood on the basis of our density measuring method plus the slow, frozen relaxation in PCP that seems to cause a crossover effect. The density has been averaged in the infected regions, where frozen ‘‘islands’’ can appear that do not evolve at all but keep the non-natural densities for long times. Therefore we overestimate the size of the region in which densities really relax. To verify this picture we have performed simulations where we averaged the 1's and 2 over fixed size. In this case we started the PCP process from ran-

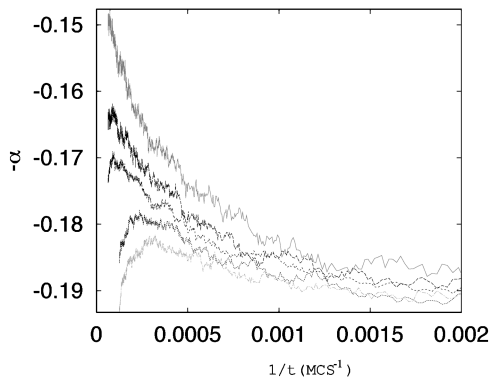


FIG. 2. The same as Fig. 1 in the case of $\rho_1(0)=0.432$ and $p=0.07715, 0.0771, 0.07705, 0.077, 0.0769$ (from bottom to top curves). Scaling can be observed for $p=0.0770(5)$, with the exponent $\alpha(t\rightarrow\infty)\approx 0.16$.

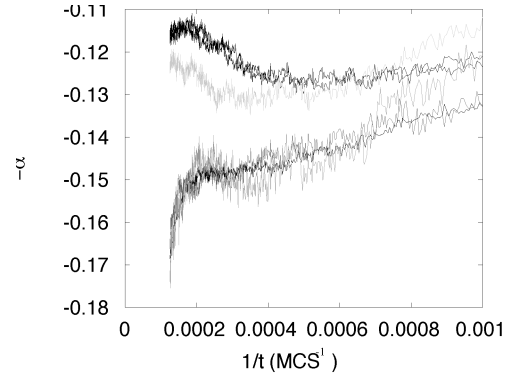


FIG. 3. Local slopes for $\Delta\rho_1(t)$ in TTP simulations for $p=0.6887$ (top curves), $p=0.6897$ (bottom curves); and $\rho_1(0)=0.4, 0.6894, 0.8$. Scaling can be observed for approximately $p=0.6894$, with the exponent $\alpha(t\rightarrow\infty)\approx 0.16(1)$.

domly distributed pairs in a system of size $L=16000$ and followed the evolution up to $t_{\text{MAX}}=16000$. In the cases of $\rho_1(0)=0$ and $\rho_1(0)=0.33$ we found that the critical point is approximately $p_c=0.07708$ nearer to the $P(t)$ and the $N(t)$ DS results. The scaling exponent was again DP like: $\alpha\approx 0.16$.

In the case of the TTP model we have performed simulations for $\rho_1(0)=0.4, \rho_1(0)=0.6894$ and $\rho_1(0)=0.8$ initial particle densities. In this model, we did not observe any shift of p_c , as Fig. 3 shows, and all the densities scale with the $\beta/\nu_{\parallel}=0.1596(4)$ exponent.

The similar power-law behaviors of $\rho_2(t)$ and $\Delta\rho_1(t)$ can be understood if one considers the coupled Langevin equations describing the time evolution of the processes. For example, in the case of the PCP model, they look like [6]

$$\frac{\partial\phi_1(x,t)}{\partial t} = c_1\nabla_x^2\phi_2 + r_1\phi_2 - u_1\phi_2^2 - w_1\phi_1\phi_2 + \dots + \eta_1(x,t), \quad (8)$$

$$\frac{\partial\phi_2(x,t)}{\partial t} = c_2\nabla_x^2\phi_2 + r_2\phi_2 - u_2\phi_2^2 - w_2\phi_1\phi_2 + \dots + \eta_2(x,t), \quad (9)$$

where $\eta_1(x,t)$ and $\eta_2(x,t)$ are Gaussian uncorrelated noise terms proportional to $\sqrt{\phi_2}$. One can see that the equations are coupled strongly by w terms; since the right-hand sides of the equations contain the same powers of the scaling fields, the time derivatives are expected to have the same scaling too.

In Ref. [6] it was argued that

$$\phi_1(t) = \phi_1^{\text{nat}} + [\phi_1(0) - \phi_1^{\text{nat}}]e^{-w_1\int_0^t\phi_2(x,s)ds} \quad (10)$$

($\phi_1^{\text{nat}}=r_1/w_1$ is the natural concentration in this formalism) may be taken as an approximate solution of Eq. (8), in which case the $\phi_1\phi_2$ cross term in Eq. (9) has the form

$$-w_2\phi_2[\phi_1(0) - \phi_1^{\text{nat}}]e^{-w_1\int_0^t\phi_2(x,s)ds}. \quad (11)$$

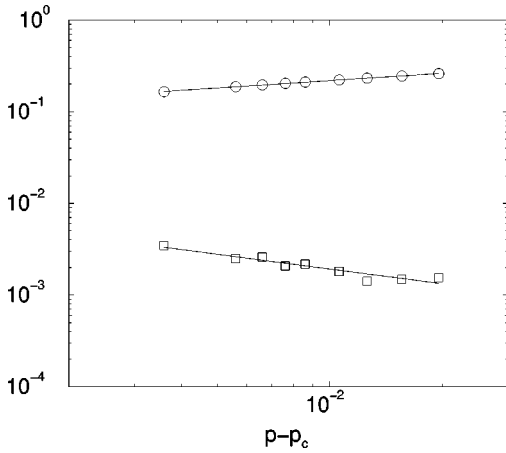


FIG. 4. Log-log plot of $\rho_1(p) - \rho_1(p_c)$ (\circ) and respective fluctuation (\square) vs $p - p_c$ above the critical point of the TTP model. Least-square regression results in $\beta = 0.27(2)$ and $\gamma = 0.53(6)$. The averaging was done following 45 000 initialization MC steps over 5000 time steps and 300 trials.

The power-law time dependence of ϕ_1 is obvious, because an exponential relaxation to ϕ_1^{nat} would just give in Eq. (9) a term similar to $r_2\phi_2$ that can shift the critical point but not the critical indices (from the DP values).

The long-range scaling behavior of the nonorder density suggests that the ϕ_1 field possesses critical fluctuations. To test this, we have performed steady state simulations as well. We have measured the 0, 1, and 2 densities in the case of the TTP model just above the critical point. We considered $L = 4000$ systems and let them evolve from random initial conditions with p slightly above $p_c = 0.6894$; approximately 40 000 MC lattice updates were necessary to reach the steady state. As one can see in Fig. 4, least-square fits of $\ln[\rho_1(p) - \rho_1(p_c)]$ vs $\ln(p - p_c)$ resulted in regular DP scaling exponent $\beta = 0.27$ [16]. The other two densities (0's and 2's) exhibited the same steady state exponents too. For the fluctuation of $\Delta\rho_1 \equiv \rho_1(p) - \rho_1(p_c)$ we observed the scaling

$$\langle \Delta\rho_1^2 \rangle - \langle \Delta\rho_1 \rangle^2 \propto |p - p_c|^{-\gamma} \quad (12)$$

with the $\gamma = 0.53(6)$ exponent, which agrees with the DP universality class value again [19].

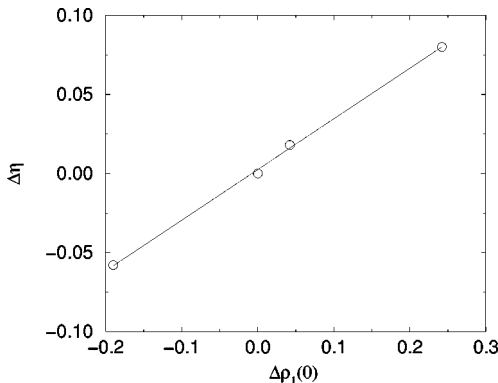


FIG. 5. Initial concentration dependence of the exponent η for the PCP model. Linear regression gives a slope $0.320(7)$ between $\eta - \eta_{DP}$ and $\rho_1(0) - \rho_1^{nat}$.

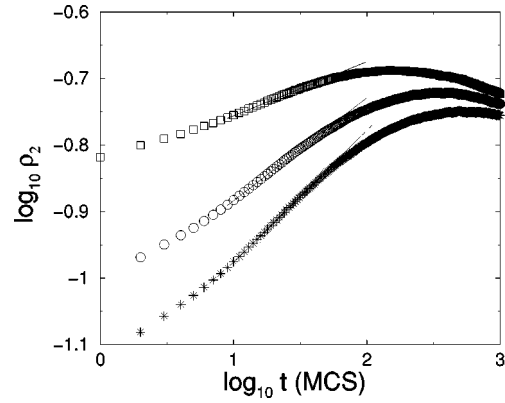


FIG. 6. Double-logarithmic plot of ρ_2 vs t for the critical TTP in early times for $\rho_2(0) = 0.02$ and $\rho_1(0) = 0.4, 0.69,$ and 0.8 (top to bottom curves). The slopes of the straight lines are $0.08, 0.15,$ and 0.19 (respectively).

The critical behavior of the ϕ_1 field demands that extra care is taken when dealing with truncated versions of Eqs. (8) and (9). A numerical integration of Eq. (9), including the non-Markovian term (11), was carried out by Lopez and Muñoz [20] and revealed that the presence of the memory term is responsible for scaling up to some time with nonuniversal values of η and δ . However, these authors did not find the linear relation between the shift from the DP values ($\eta - \eta_{DP}$ and $\delta - \delta_{DP}$) and $\phi_1(0) - \phi_1^{nat}$ that our results for PCP show (Fig. 5) and was also found in previous TTP studies [18]. We think one has to take into account the omitted terms in Eq. (10), which we have shown to exhibit a power law in time and therefore give a relevant contribution to the renormalization of ϕ_2 .

We have also investigated the early stages of the relaxation of the order parameter. In Ref. [6] it was suggested that in the short-time regime ($w_1\phi_2t \ll 1$) one might observe the dynamic percolation scaling of Grassberger [4]. Early time scaling—currently referred to as *critical initial slip*—was introduced by Jansen, Schaub, and Schmittman [21] and recently investigated by van Wijland, Oerding, and Hilhorst [22] for a reaction-diffusion model with two kinds of particles, using renormalization-group (RG) analysis. They found nonuniversal dependence (on the initial particle distribution) of the slip exponent in the case of unequal diffusion coefficients.

The system was prepared by “adding” a few 2's [uniform density $\rho_2(0) \ll 1$] to a random uniform background of 0's and 1's [density $\rho_1(0)$]. The system evolution was recorded up to $t = 1000$ MCS and averages were performed over independent runs (10^4 typically). Log-log plots of

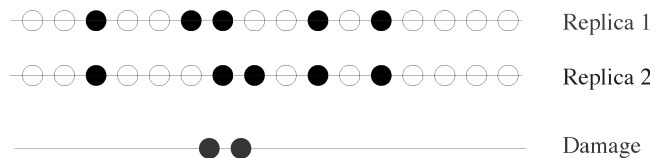


FIG. 7. Sample initial state of the PCP DS simulations. Pairs (seeds) are displaced in the middle of the lattices of replicas with single space shift that generates two damage variables. In the case of the TTP DS simulations the initialization is the same, except that we have 2's instead of pairs.

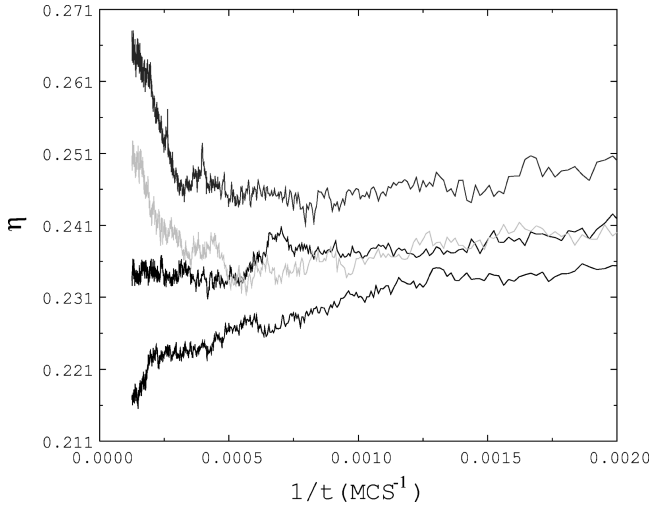


FIG. 8. Local slopes $\eta(t)$ near the PCP DS transition point, for $\rho_1(0)=0$ and $p=0.077\,08, 0.077\,04, 0.077, 0.0769$ (from bottom to top). The DS critical point is at $p_d=0.077\,04$ with the corresponding exponent $\eta=0.234(3)$.

$\rho_2(t)$ —see Fig. 6—show a linear region [lasting for $10 < t < 100$ for $\rho_2(0)$ equal to a few percent] with a slope that is independent of $\rho_2(0)$ but depends on $\rho_1(0)$. This may be evidence for the critical initial slip with a nonuniversal slip exponent θ' equal to that slope. Unlike the DP case, where $\theta' = \eta$ [22], the values we found for θ' differ significantly from the corresponding η values.

IV. DAMAGE SPREADING SIMULATIONS

The damage spreading simulations have been initialized by two replicas of states with identical, but random uniform distribution of single 1's of given concentrations. Then a seed (a pair in the case of PCP and a 2 in the case of TTP) is added to each replica such that they become nearest neighbors and the initial difference is 2 (see Fig. 7). The order parameter characterizing the damage is the Hamming distance between replicas,

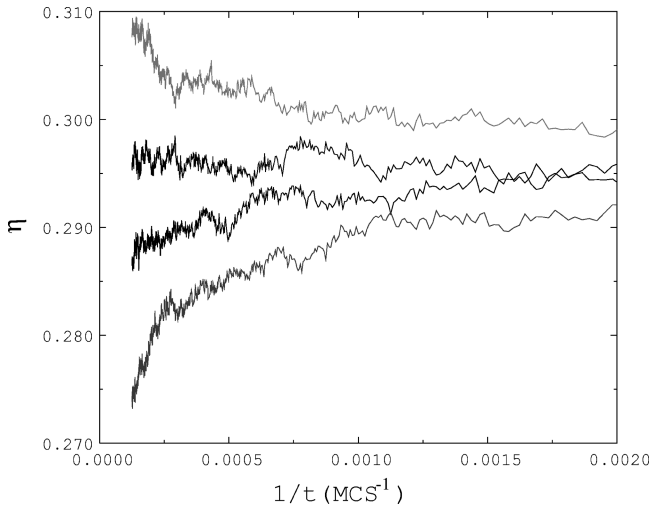


FIG. 9. Same as Fig. 8 for $\rho_1(0)=0.2$ and $p=0.077\,16, 0.077\,12, 0.077\,08, 0.077\,04$ (bottom to top). The DS critical point is at $p_d=0.077\,08$ with the corresponding $\eta=0.296(1)$.

TABLE I. Damage spreading simulation results in the PCP model. The numbers in parentheses are numerical errors.

$\rho_1(0)$	p_d	η_{DS}	δ_{DS}	z
0.0	0.077 04	0.234(3)	0.24(2)	1.23(3)
0.2	0.077 08	0.296(1)	0.18(1)	1.24(4)
0.242	0.077 09	0.314(6)	0.16(9)	1.24(5)
0.432	0.077 14	0.372(5)	0.11(1)	1.26(3)

$$D(t) = \left\langle \sum_{i=1}^L |s(i) - s'(i)| \right\rangle, \quad (13)$$

where $s(i)$ denote the pairs in the case of PCP and the variable 2's in the case of the TTP model. At the DS critical point (p_d) we expect that the order parameter scales as

$$D(t) \propto t^\eta. \quad (14)$$

Similarly the survival probability of damage variables behaves as

$$P(t) \propto t^{-\delta} \quad (15)$$

and the average mean square distance of damage spreading from the center scales as

$$R^2(t) \propto t^z. \quad (16)$$

Averages were performed over $N_s = 10^6$ independent runs for each value of p in the vicinity of p_d [but for $R^2(t)$ only over the surviving runs]. The t_{MAX} was 8000 in these simulations. Figures 8, 9, 10, and 11 show the local slopes results of the Hamming distance for the PCP model and different initial concentration of 1's. The p_d transition points are found to coincide with the ordinary critical points of the replicas

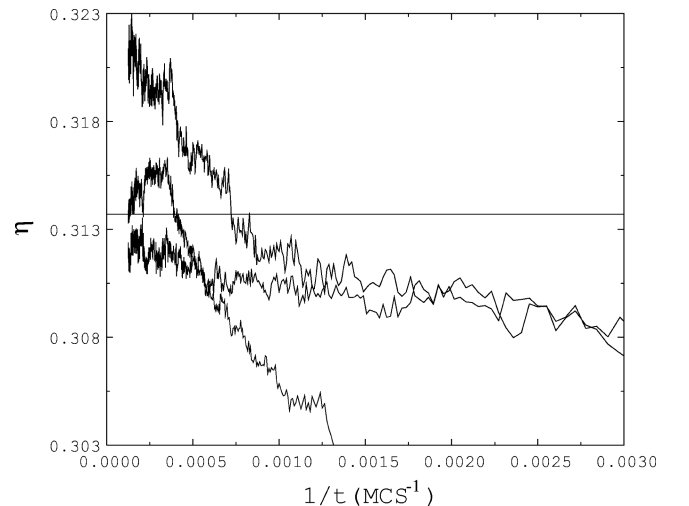


FIG. 10. Same as Fig. 8 for $\rho_1(0)=0.242$ and $p=0.077\,09, 0.077\,07$ (from bottom to top). The dotted curve corresponds to simulations with system generated initial configurations and $p = 0.077\,09$. The DS critical point is at $p_d=0.077\,09$ with the corresponding exponent $\eta=0.314(6)$. The dashed line shows the estimated value of the DP exponent obtained by simulations.

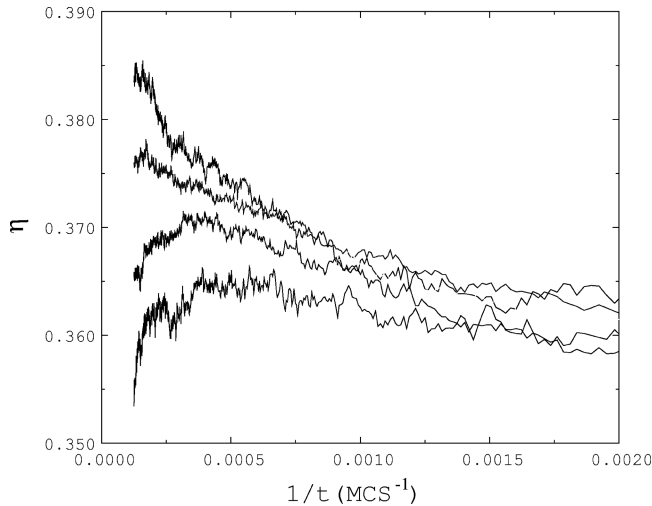


FIG. 11. Same as Fig. 8 for $\rho_1(0)=0.432$ and $p=0.077\ 18, 0.077\ 16, 0.077\ 14, 0.077\ 12, 0.077\ 1, 0.077\ 1$ (from bottom to top). The DS critical point is at $p_d=0.077\ 14$ with the corresponding exponent $\eta=0.372(5)$.

within numerical accuracy. A small, but monotonic tendency in the variation of p_d (as in case of p_c) with initial conditions can be observed in Table I.

The DS critical η and δ exponents show the same non-universal behavior as the corresponding ordinary critical exponents (see also [15,17]) and coincide with them within numerical precision. The exponent z is constant within numerical accuracy.

In the case of $\rho_1(0)=0.242$, we performed runs with uniform initial distributions and with system generated configurations of isolated 1's with the same average concentration. The latter, nonuniform distribution was generated by letting a single replica to run at $p=0.077\ 09$ until it reached the absorbing state; then the infected area of the system was used as initial state for DS simulations, similarly to what was done in Ref. [1]. We do not see significant differences (Fig. 10) between the two cases for t large, both of them result in exponents in agreement with the best DP class $\eta_{DP}=0.3137$ value [19].

In the case of the TTP model we performed DS simulations for $\rho_1(0)=0.4$ only. We could see analogous DS behavior as in the case of the PCP. Again the critical point and exponents coincide with the corresponding critical values. Similarly to what has been observed in models that belong to the PC universality class [13], we can also conclude that if the DS transition point coincides with p_c the scaling behavior is inherited.

An interesting implication of this result can be stated exploiting the possible mapping of these models to self-organized critical (SOC) models [15]. The corresponding critical sandpile models are not chaotic in the sense that the avalanches (or clusters) arising from the dropping of pairs (or seeds) to the lattice result in trajectories with power-law increasing differences only. This does not exclude the possibility that other SOC models generated by the way of Ref.

[15] are chaotic, since if p_d happens to be in the active phase, the perturbations in the SOC model generate differences that increase faster than a power law.

V. CONCLUSIONS

Two representatives of systems with a continuous phase transition to infinitely degenerate absorbing states (PCP and TTP models) have been investigated numerically in $1-d$. In order to clarify the influence of the initial condition on the dynamic properties, we have performed time-dependent simulations and analyzed the evolution of both the order parameter and the nonorder field densities. We gave numerical evidence that the nonorder field is in a critical state simultaneously with the order-parameter field. The isolated particles density exhibits a continuous phase transition with DP exponents to a nonabsorbing state, therefore its fluctuations cannot be neglected when one tries to understand the nonuniversal behavior of critical exponents. Due to the dynamic coupling between the two fields, the slow (power-law) decay of the background particle density induces a long-time memory of the susceptibility to spreading of the order parameter [4].

We have found that the estimates of p_c obtained from the time dependence of the survival probability are consistent with those given by DS studies; a small shift as a function of $\rho_1(0)$ is exhibited, which we interpret as due to the slowly decaying memory [4]. The study of the density in seed-growing simulations produced values of p_c slightly off the former ones; we think this is probably a crossover effect.

We do not see such a p_c shift for the TTP model so this maybe specific of the PCP model, connected to the nonergodicity of its absorbing states. In both models the critical exponents η and δ have been found to behave linearly as a function of the initial particle concentration.

Preliminary studies of the early time critical regime suggest the existence of a (short) initial slip regime characterized by an exponent that depends on the initial particle concentration $\rho_1(0)$, in agreement with RG predictions for a similar model [22]. This topic needs, however, additional investigation.

The damage spreading investigations have shown that the DS point coincides with the critical point and so the critical indices “inherit” a nonuniversal scaling behavior similar to what was found in an earlier study [13].

We hope that our study will stimulate further field theoretical analysis of critical spreading in systems with many absorbing states.

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