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Series expansion solution of the Wegner–Houghton renormalisation group equation

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Abstract. The momentum independent projection of the Wegner-Houghton renormalisation group equation is solved with power series expansion. Convergence rate is analyzed for the n-vector model. Further evidence is presented for the first order nature of the chiral symmetry restoration at finite temperature in QCD with 3 light flavors.

1 Introduction

The change of the ground state of quantum chromodynamics as a function of the temperature and/or the density belongs to the most intensively studied non-perturbative phenomena. The direct approach is to explore the variation of the characteristic order parameter(s) by Monte Carlo simulation of the full gauge-matter system.

The order parameters of the euclidean theory are Lorentz-scalar quantities with eventual index structure related to the flavor symmetries. They are built from the original quark and gluon fields. The simplest example is the Polyakov loop, which is a flavor singlet quantity:

$$L(\underline{x}) \equiv \operatorname{Tr} P \exp\left\{ i \int_0^\beta A_0(\underline{x}, x_0)\, dx_0 \right\}, \tag{1}$$

where $\beta = (k_B T)^{-1}$. It breaks the global $Z(N)$ invariance of the pure $SU(N)$ gauge action and measures the excess free energy of a static coloured source over the sourceless vacuum.

The other representative example is the tensor of the quark condensate

$$M^{ij}(\underline{x}) = \bar{\psi}^i_L(\underline{x}) \psi^j_R(\underline{x}), \quad i, j = 1, \ldots, N_{\text{flavor}} \tag{2}$$

transforming in the fundamental representation of $SU(N_f)_L \otimes SU(N_f)_R$. The dimension of the space where the field theory of the order parameter is embedded into coincides at finite temperature with the spatial extension of the system.

With these informations one builds the most general theory for the order parameter in question ($\Phi_\alpha(x)$):

$$S = \sum_i^\infty \prod_{j=1}^i \int d^d k_j A(k_1, \ldots, k_i)^{\alpha, \beta \cdots} \Phi_\alpha(k_1) \Phi_\beta(k_2) \ldots, \tag{3}$$

where $\alpha, \beta \cdots$ stand for the "flavor" index and the action is displayed in the momentum space.

The fixed point structure of the coupling space of (3) provides information concerning the stable phases (absolute attractive points) and the number of couplings to be tuned to realize a specific transition (degree of instability of the fixed point governing the transition). Temperature driven continuous transitions are controled by fixed points with a single unstable (relevant) direction. In its absence transitions which can be reached by varying a single parameter (usually the temperature) turn out to be of first order. These basic facts were used to perform the universality classification of the deconfining transition described by the order parameter (1) [1].

In most models the phase structure of the general effective theory is very rich. This circumstance prevents one to draw definite conclusions without identifying the actual fixed point where the couplings of the effective theory explicitly derivable from QCD flow to.

Explicit computations were performed for the deconfinement transition in the strong coupling regime of the gauge models [2] and the fixed points of the arising effective theory were studied in the Migdal-Kadanoff approximation [3] and also with Monte Carlo renormalisation group [4]. For the chiral symmetry restoration $d = 4$-ε dimensional leading ε-order calculations are available [5].

In a broader framework important issues were raised concerning the fixed point structure of various field theories: ϕ^4-model in $d = 4$ [6], the Higgs-