

# Critical Synchronization Dynamics of the Kuramoto Model on a Large Human Connectome

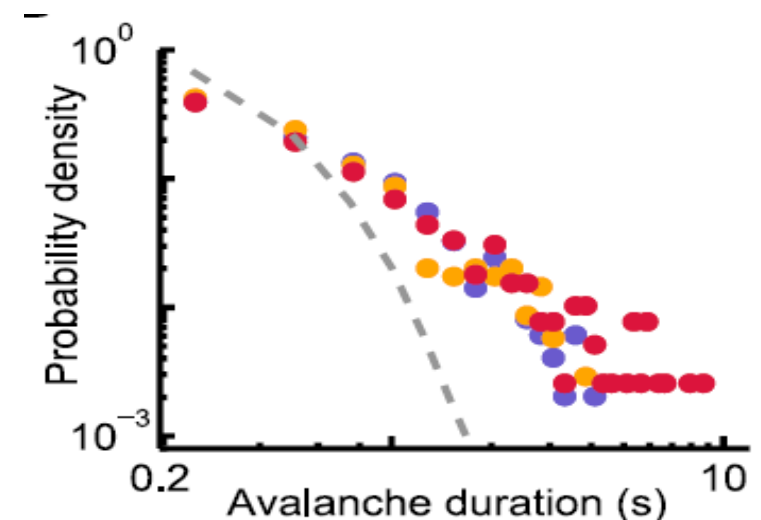
Géza Ódor EK-MFA Complex Systems Department, Budapest

Jeffrey Kelling HZDR Dresden, Gustavo Deco UPF Barcelona

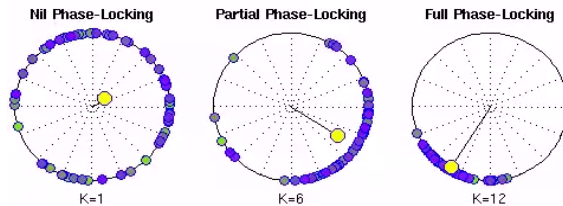
Theoretical research and experiments suggest that the brain operates at or near a **critical state** between sustained activity and an inactive phase, exhibiting optimal computational properties (see: *Beggs & Plenz J. Neurosci. 2003; Chialvo Nat. Phys. 2010; Haimovici et al. PRL 2013*)

Individual neurons emit periodic signals:  
(*Y. Penn et al PNAS 113 (2016) 3341*)

→ Critical behavior at the **synchronization transition?**



# Kuramoto oscillator model (1975)



Nil, partial and full phase-locking in an all-to-all network of Kuramoto oscillators. Phase-locking is governed by the coupling strength  $K$  and the distribution of intrinsic frequencies  $\omega$ . Here, the intrinsic frequencies were drawn from a normal distribution ( $M=0.5\text{Hz}$ ,  $SD=0.5\text{Hz}$ ). The yellow disk marks the phase centroid. Its radius is a measure of coherence.

$$\dot{\theta}_i(t) = \omega_{i,0} + \frac{K}{k_i} \sum_j W_{ij} \sin[\theta_j(t) - \theta_i(t)]$$

phases  $\theta_i(t)$  in-degrees  $k_i$

global coupling  $K$  is the control parameter

weighted adjacency matrix  $W_{ij}$

$\omega_{i,0}$  is the intrinsic frequency of the  $i$ -th oscillator,

$$R(t) = \frac{1}{N} \left| \sum_{j=1}^N e^{i\theta_j(t)} \right|$$

Order parameter : average phase:

Non-zero, above critical coupling strength  $K > K_c$ ,

Evolves to zero for  $K \leq K_c$

At  $K = K_c$  from incoherent initial state : initial growth:  $R \sim t^\eta$

Critical synchronization transition for  $D > 4$  spatial dimensions, which is mean-field like: i.e.  $D \rightarrow \infty$  (full graph)

The dynamical behavior suffers very strong corrections to scaling and *chaoticity*, see:

Róbert Juhász, Jeffrey Kelling and Géza Ódor:

Critical dynamics of the Kuramoto model on sparse random networks

J. Stat. Mech. (2019) 053403

# Large Human Connectome graphs

Diffusion and structural MRI images with  
 $1 \text{ mm}^3$  voxel resolution :  
 $10^5 - 10^6$  nodes

Hierarchical modular graphs

Top level: 70 brain region (Desikan atlas)

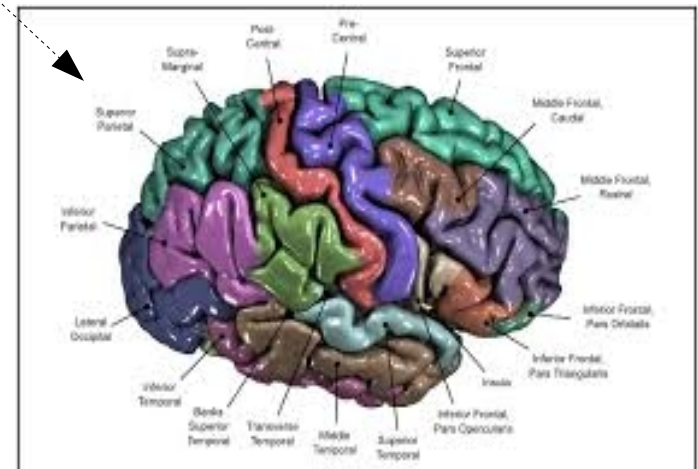
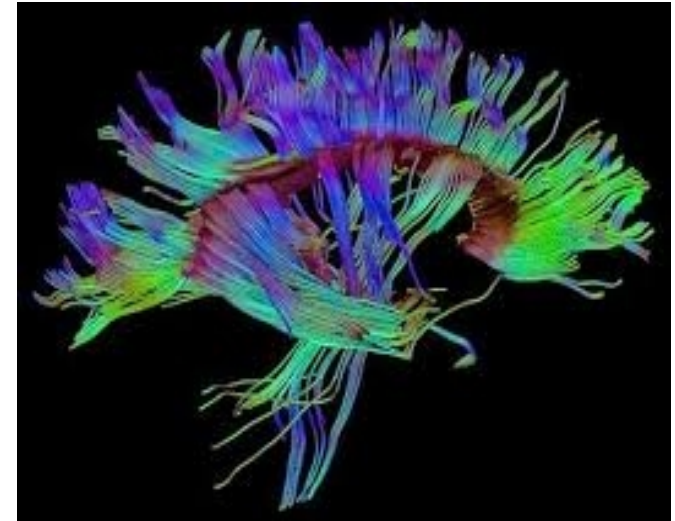
Lower levels obtained by deterministic  
tractography: FACT algorithm

Map : voxel  $\rightarrow$  vertex ( $\sim 10^7$ )

fiber  $\rightarrow$  edge ( $\sim 10^{10}$ )

+ noise reduction  $\rightarrow$  graph

undirected, weighted



Structural graphs of nodes (containing  $\sim 10^4$  neurons) and power-law weight distributed edges see : [Michael T. Gastner and Géza Ódor, Scientific Reports 6 \(2016\) 27249](#)

# Kuramoto solution for the KKI-18 graph with $N=804\,092$ nodes and $41\,523\,908$ weighted edges

The synchronization transition point determined by growth from disorder

KKI-18 has  $D = 3.05 < 4 \rightarrow$

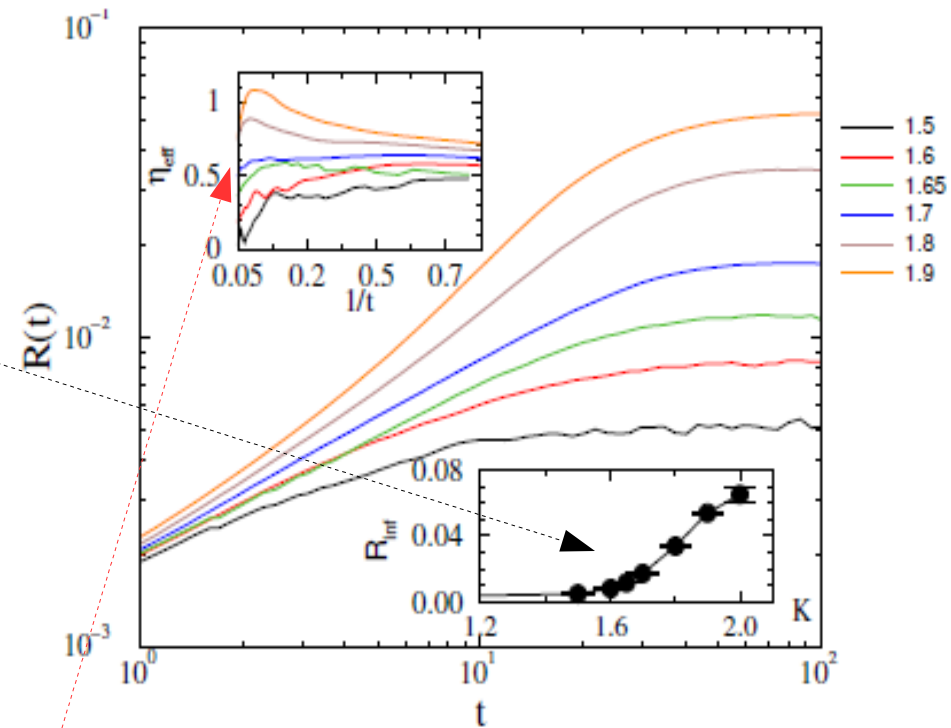
Smooth crossover to partial synch.

Fat-tailed link weight distribution, incoming weight normalization is applied:

$$W'_{i,j} = W_{i,j} / \sum_{j \in \text{neighb. of } i} W_{i,j}$$

to provide local homeostasis  
(suppress hubs)

$K_c = 1.7$  and growth exponent:  $\eta = 0.6(1)$



# Determination of the characteristic time exponent: $\tau_t$

Measure characteristic times  $t_x$  of first dip below:  $R_c = \sqrt{1/N}$

Runs for  $\sim 10,000$  random, independent  $\omega_i$  realizations

Histogramming of  $t_x$  at  $K_c = 1.6$

Critical exponent:  $\tau_t = 1.2 (1)$

Below the transition point :  $K < 1.6$

non-universal power laws in the range of experiments of activity durations :

$1.5 < \tau_t < 2.4$  (Palva et al 2013)

Géza Ódor and Jeffrey Kelling :

Critical synchronization dynamics of the Kuramoto model on connectome and small world graphs Scientific Reports 9 (2019) 19621

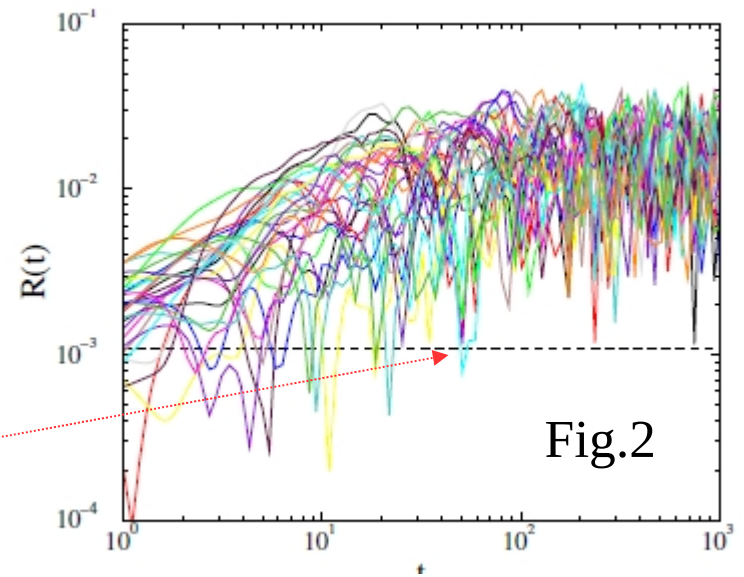


Fig.2

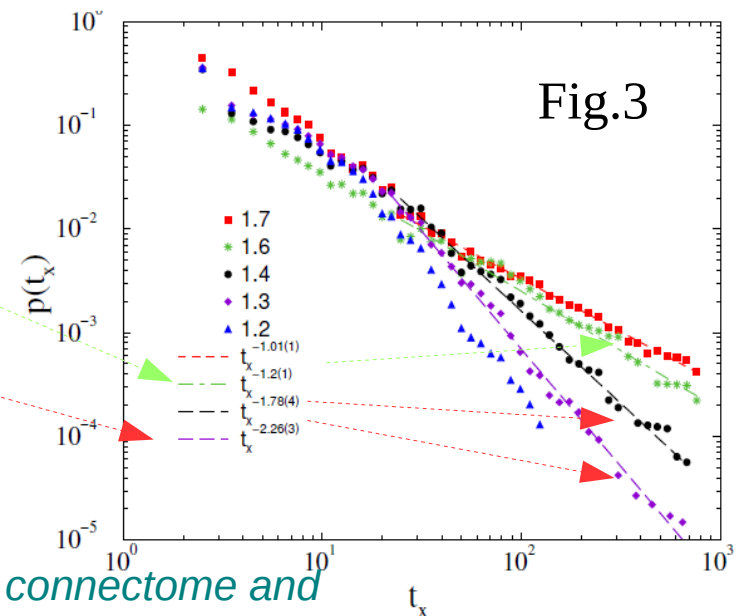


Fig.3

# The effect of additive stochastic noise

Brain experiments:  $\omega_i > 0$ , distributions are narrow:  $\sigma_i \sim 0.02$   
and have mean value:  $\langle \omega_i \rangle \sim 0.05$

## Galilean invariance of the Kuramoto equation:

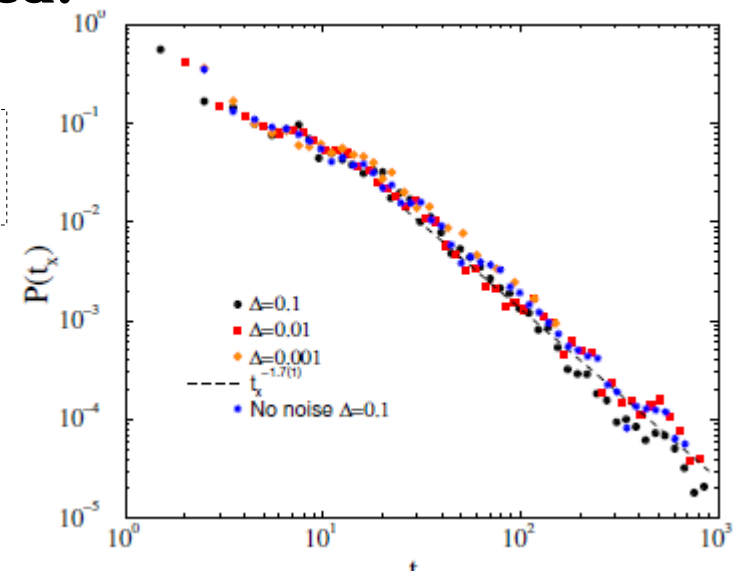
$\langle \omega_i \rangle \neq 0$  can be gauged out by a rotating coordinate system and rescaling of  $\omega_i$  as :  $\omega_i \rightarrow a\omega_i'$   $t \rightarrow (1/a)t'$   $K \rightarrow aK'$

**Existing results can be obtained at later times and weaker K**

Gaussian distributed annealed noise is added:

$$\dot{\theta}_i(t) = \omega_{i,0} + K \sum_j W_{ij} \sin[\theta_j(t) - \theta_i(t)] + s\xi(i)$$

**Negligible effect of the weak noise**



# Conclusions

Heterogeneity effects are simulated on **large** human connectomes

This enables us to distinguish from finite size rounding effects

Local homeostasis is needed to find criticality

New method to determine crossover to desynchronization and to measure the characteristic times

Below the transition point non-universal dynamical scaling of synchronization → Frustrated synchronization in modules

Desynchronization durations exponents  $\tau_t$  agree in vivo activity experiments for humans

Results are valid for realistic, narrow band  $\omega_i$  frequency distributions

Insensitivity for weak, additive Gaussian noise

*G. Ódor, J.Kelling, G. Deco: accepted in J. Neurocomputing*

*preprint : arXiv:1912.06018*