Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

LETTER

Local scale invariance in the parity conserving non-equilibrium kinetic Ising model

Géza Ódor

Research Institute for Technical Physics and Materials Science, H-1525 Budapest, PO Box 49, Hungary
E-mail: odor@mfa.kfki.hu

Received 30 June 2006
Accepted 5 October 2006
Published 30 November 2006

Online at stacks.iop.org/JSTAT/2006/L11002
doi:10.1088/1742-5468/2006/11/L11002

Abstract. Local scale invariance has been investigated in the non-equilibrium kinetic Ising model exhibiting an absorbing phase transition of parity conserving type in $1+1$ dimensions. Numerical evidence has been found for this symmetry and estimates for the critical ageing exponents are given.

Keywords: classical Monte Carlo simulations, critical exponents and amplitudes (theory), finite-size scaling, phase transitions into absorbing states (theory)

ArXiv ePrint: cond-mat/0606724

J. Stat. Mech. (2006) L11002

 1742-5468/06/L11002+7$30.00

Contents

J. Stat. Mech. (2006) L11002

1. Introduction

The classification of the universality classes of non-equilibrium phase transitions is still an open problem of statistical physics [1]–[4]. In equilibrium, conformal invariance (CI) [5]–[7] enables this in two-dimensional critical systems as the consequence of a larger group (the CI group) than the mere scale transformations. Recently the generalization of the generators of CI (albeit without invariance under time translations) was proposed for anisotropic, dynamical models [8, 9]. The corresponding invariance is the so-called local scale invariance (LSI). Since it is supposed to be the extension of the dynamical scale transformations for such systems it may serve as a convenient tool for classifying universality classes of non-equilibrium systems as well.

The quantities of main interest are the two-time autocorrelation function $C(t, s)$ and the autoresponse function $R(t, s)$, which describe ageing phenomena (for recent reviews see [10])

$$C(t, s) = \langle \phi(t, \vec{r}) \phi(s, \vec{r}) \rangle \tag{1}$$

$$R(t, s) = \left. \frac{\delta \langle \phi(t, \vec{r}) \rangle}{\delta h(s, \vec{r})} \right|_{h=0} = \left\langle \phi(t, \vec{r}) \widetilde{\phi}(s, \vec{r}) \right\rangle \tag{2}$$

where ϕ and $\widetilde{\phi}$ are the fields in the Janssen–de Dominicis formalism [11, 12] and h is the magnetic field conjugate to ϕ. For $t, s \to \infty$ and $y = t/s > 1$ one expects the scaling forms

$$C(t, s) = s^{-b} f_{\mathrm{C}}(t/s) \tag{3}$$

$$R(t, s) = s^{-1-a} f_{\mathrm{R}}(t/s), \tag{4}$$

where a and b are ageing exponents and f_{C} and f_{R} are scaling functions such that $f_{\mathrm{C,R}}(y) \sim y^{-\lambda_{\mathrm{C,R}}/Z}$ for $y \gg 1$. Here λ_{C} and λ_{R} are the autocorrelation [13] and autoresponse [14] exponents respectively and independent of equilibrium exponents and the dynamical exponent Z (defined as usual: $Z = \nu_{\|}/\nu_{\perp}$).

As in the case of CI one expects LSI to fully determine the functional form of the scaling functions. Henkel *et al* derived $R(t, s)$ in general and the form of $C(t, s)$ for $Z = 2$ by identifying the quasi-primary operators of the theory [15, 16]. The generalized form of $R(t, s)$ takes into account the difference between physical observables defined in lattice models and the associated quasi-primary scaling operators of the underlying field theory as well. This ansatz looks as follows:

$$R(t,s) = s^{-1-a} \left(\frac{t}{s}\right)^{1+a'-\lambda_{\rm R}/Z} \left(\frac{t}{s}-1\right)^{-1-a'}, \tag{5}$$

where $a' \neq a$ is an independent ageing exponent in general. Some systems with detailed balance symmetry have been analysed recently and found to satisfy (5) [9], [18]–[23] with $a \neq a'$. On the other hand renormalization-group results for some important universality classes concluded that $a = a'$ should hold. In particular, explicit two-loop field-theoretical computation of $R(t,s)$ for the $O(N)$ universality class and model A dynamics at the critical point claim $a = a'$ [24, 25].

Recently numerical simulations of the non-equilibrium contact process (CP) did not satisfy that form completely [26] and Hinrichsen argued that LSI is not a generic property of ageing phenomena but is restricted to diffusive models ($Z = 2$) or above the upper critical dimension. In [16] Henkel *et al* suggested that there is crossover in the case of non-equilibrium critical dynamics because both the ageing regime ($t - s \sim O(s)$) and the quasi-stationary regime ($t - s \ll s$) display scaling, with the same length scale $L(t) \propto t^{1/Z}$. For a more detailed discussion of these results see a very recent review [17].

In this letter I present simulation results for another non-equilibrium critical model, the parity conserving (PC) non-equilibrium Ising model (NEKIM) in $1+1$ dimensions. I provide numerical evidence that in this model $C(t,s)$ and $R(t,s)$ can be fitted with the forms equations (3), (5); hence this non-equilibrium critical model exhibits LSI scaling invariance.

2. The PC class NEKIM model

The NEKIM model was introduced and analysed first by Menyhárd [27] as a generalization of the kinetic Ising model [28] by adding spin-exchange updates in between the spin-flip steps of the Glauber Ising model. In one dimension the domain walls (kinks) between up and down regions can be considered as particles. The spin-flip dynamics can be mapped onto particle movement:

$$\uparrow\downarrow\downarrow \overset{w_i}{\rightleftharpoons} \uparrow\uparrow\downarrow \sim \bullet\circ \rightleftharpoons \circ\bullet \tag{6}$$

or onto the annihilation of neighbouring particles:

$$\uparrow\downarrow\uparrow \overset{w_o}{\rightarrow} \uparrow\uparrow\uparrow \sim \circ\circ \rightarrow \bullet\,\bullet\,. \tag{7}$$

Therefore the $T = 0$ Glauber dynamics is equivalent to the annihilating random walk (ARW). This is a doubly degenerate phase; an initial state decays algebraically to the stationary state, which is one of the absorbing ones (all spins up or all spins down, provided the initial state has an even number of kinks). By mapping the spin-exchange dynamics in the same way more complicated particle dynamics emerges, for example,

$$\uparrow\uparrow\downarrow\downarrow \overset{p_{\rm ex}}{\rightleftharpoons} \uparrow\downarrow\uparrow\downarrow \sim \circ\bullet\circ \rightleftharpoons \bullet\bullet\bullet \tag{8}$$

and one particle may give birth of two others or three particles may coagulate to one. Therefore this model is equivalent to branching and annihilating random walks with an

doi:10.1088/1742-5468/2006/11/L11002

J. Stat. Mech. (2006) L11002

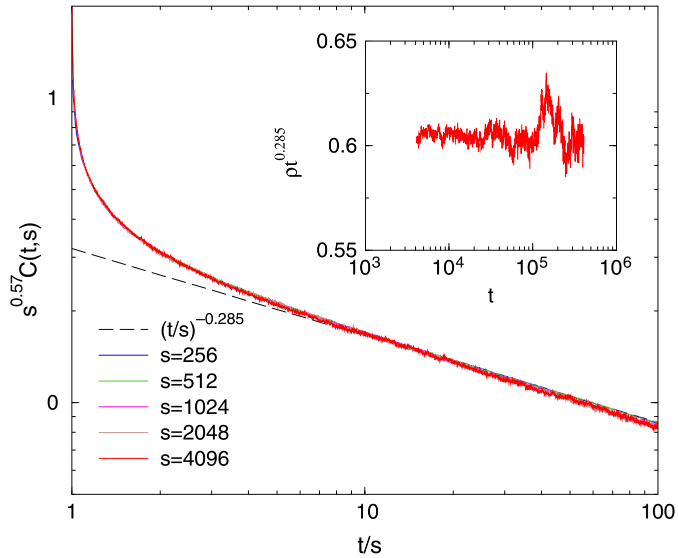

Figure 1. Autocorrelation $C(t, s)$ of the critical NEKIM model for several values of waiting time s as a function of the scaling variable t/s. The dashed line has the slope $-0.285 \sim -\alpha$. The inset shows the kink density decay $\rho(t)^{0.285}$ up to $t = 409\,600$ MCS in a system of size $L = 10^5$.

even number of offspring [30, 31]. On increasing the spin exchange a second-order phase transition takes place [27] for the *kinks* from an absorbing to an active state, which belongs to the parity conserving (PC) universality class [29, 31, 32].

In [36] this model has been investigated using high precision cluster simulations with the parametrization

$$p_{\mathrm{ex}} = 1 - 2\Gamma \tag{9}$$

$$w_i = \Gamma(1 - \delta)/2 \tag{10}$$

$$w_o = \Gamma(1 + \delta) \tag{11}$$

originating from the Glauber Ising model [28]. In this work I present simulations at the critical point determined in previous works [27], [33]–[36]. The parameters chosen are: $\Gamma = 0.35$, $p_{\mathrm{ex}} = 0.3$, $\delta_{\mathrm{c}} = -0.3928(2)$. Here the kink ($n_i \in (0, 1)$) density decays as

$$\langle n_i(t)\rangle \propto t^{-0.285(2)} \tag{12}$$

as can be seen in the inset of figure 1. In previous works the NEKIM algorithm was introduced as follows. The spin-flip part was applied using two-sublattice updating. Following this, states of the spins were stored and L (L is the size of the system) random attempts at spin exchanges were made using the stored situation of states of the spins before updating the whole lattice. All these procedures together were counted as one Monte Carlo time step (MCS) of updating (throughout this work, time is measured in MCS).

J. Stat. Mech. (2006) L11002

3. Simulations

Time dependent simulations were performed in $L = 2\times10^4$–10^5 sized systems with periodic boundary conditions. The runs were started from a fully ordered kink state ($n_i(0) = 1$), i.e. an alternating up–down spin configuration ($s_i \in (-1, 1)$). I followed the quench towards the critical state and measured the kink (order parameter) density

$$\langle n_i(t)\rangle \propto t^{-\alpha}, \tag{13}$$

the kink–kink autocorrelation

$$C(t, s) = \langle n_i(t)n_i(s)\rangle, \tag{14}$$

and the autoresponse function, by flipping a spin at random site l at time s generating a kink pair

$$R(t, s) = \langle n_i(t)n_{i+1}(t)\rangle - \langle n'_i(t)n'_{i+1}(t)\rangle|_{s'_l(s):=-s_l(s)}. \tag{15}$$

The simulations were run for several values of waiting times $s = 256, 512, 1024, 2048, 4096$ and the scaled autocorrelation $C(t, s)t^{-2\alpha}$ is plotted in figure 1 with the assumption of the form of equation (3). Good data collapse (within the error margin of the simulations) in the case of systems of sizes $L = 2 \times 10^4$ could be achieved for the whole region; however for larger s and t values small deviations from the collapse could also be observed. On investigating larger systems this proved to be a finite size effect. The curve in figure 1 for $s = 4096$ shows the result of $L = 10^5$ simulations. In the asymptotic $t/s \to \infty$ limit it can be fitted by a $t^{-0.285}$ power law, corresponding to the density decay of the PC class [36]. This suggests the scaling exponents $b = 0.570(4)$, $\lambda_C/Z = 0.285(2)$. On inserting the value of the dynamical exponent of the PC class $Z = 1.75(1)$ [3] one obtains $\lambda_C = 0.498(2)$. This exponent agrees with that of the autocorrelation exponent of spins $\lambda = 1.50(2)$ [35] for $(t/s) \to \infty$:

$$\begin{aligned} A(t, s) &= \langle s_i(s)s_i(t)\rangle \\ &= f(t/s) \propto (t/s)^{-(\lambda-d+1-\eta/2)/Z}, \end{aligned} \tag{16}$$

since $\eta = 1.01(1)$, and a hyperscaling law connecting time dependent spin and kink exponents at the PC transition point derived in [34]. (The connected correlator, defined as $\Gamma(t, s) = C(t, s) - \langle n_i(t)\rangle\langle n_i(s)\rangle$, scales with the exponent $\lambda_G/Z = 1.9(1)$.)

The autoresponse function has been found to exhibit a similar nice data collapse, by plotting $R(t, s)t^{0.57}$ as a function of $y = t/s$ (figure 2). However in [26] Hinrichsen discovered that in the case of the CP model deviations from the LSI scaling form of $R(t, s)$ (5) may occur for $y \to 1$. To see this I plotted $R(t, s)s^{0.57}y^A(y - 1)^B$, suggested in [26], as a function of $y - 1$. Now one may see agreement with equation (5) if the curves fitted with the parameters collapse and are horizontal for all y values. The best agreement was achieved with $A = -1.33(1)$ and $B = -0.57(1)$, plotted in the inset of figure 2. On increasing s the observed deviations from LSI scaling for $y \to 1$ occur at smaller y values, suggesting corrections due to the microscopic reference time s. This is different from the case for the CP, where all such curves collapsed. Assuming the general form of equation (5), the fitting results in $a = -0.430(2)$, $a' = -0.43(1)$, $\lambda_R/Z = 1.9(2)$ as the dynamical exponents, with a validity of more than three decades.

J. Stat. Mech. (2006) L11002

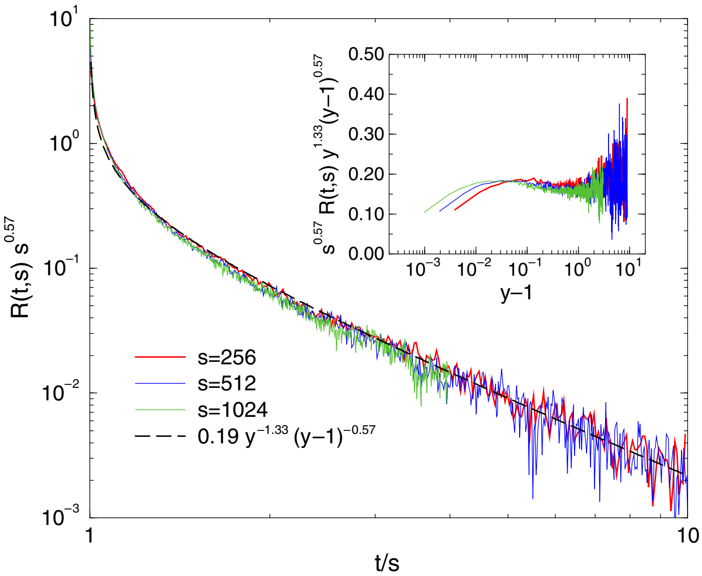

Figure 2. Autoresponse $R(t,s)$ of the critical NEKIM model for several values of waiting time s as a function of the scaling variable t/s. The dashed line is a fit with the general form (5). In the inset the rescaled autoresponse function is plotted in such a way that possible deviations from the LSI scaling are more easily observable for $y \to 1$.

4. Conclusions

In conclusion, numerical simulations of the parity conserving NEKIM model in 1D support local scale invariance at the critical point. In contrast to the contact process (belonging to the directed percolation class [1]), corrections to scaling due to the microscopic reference time vanish in the $s \to \infty$ limit. Both the autocorrelation and the autoresponse functions can well be described by the functional forms of LSI; only negligible dependence on the system sizes has been detected within the error margin of the numerical simulations. Further sources of deviations may come from the value of α and the location of the critical point. The same analysis done for $-\delta_c = 0.3925, 0.392, 0.391$ and using $\alpha = 0.286, 0.287$ did not result in visible deviations in the figures and the fitting parameters. The autoresponse function scales in such a way that $a = 2\alpha = a'$. Numerical estimates for the $\lambda_{C,R,G}$ exponents are determined and $\lambda_R = \lambda_G$ is found. This supports the conjecture of [15] that LSI can be extended to other non-equilibrium critical systems besides diffusive models ($Z = 2$) and models above the upper critical dimension.

I thank N Menyhárd, Haye Hinrichsen and M Henkel for useful discussion and A Gambassi for comments. Support from the Hungarian research fund OTKA (Grant No. T046129) is acknowledged. The author is grateful for having been given access to the NIIFI Cluster-GRID, LCG-GRID and to the Supercomputer Centre of Hungary.

References

[1] For references see: Marro J and Dickman R, 1999 *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge: Cambridge University Press)

[2] For a review see: Hinrichsen H, 2000 *Adv. Phys.* **49** 815
[3] For a recent review see Ódor G, *Phase transition universality classes of classical, nonequilibrium systems*, 2004 *Rev. Mod. Phys.* **76** 663
[4] Elgart V and Kamenev A, 2006 *Phys. Rev.* E **74** 041101
[5] Friedan D, Qiu Z and Shenker S, 1984 *Phys. Rev. Lett.* **52** 1575
[6] Cardy J L, 1987 *Phase Transitions and Critical Phenomena* vol 11, ed C Domb and J L Lebowitz (New York: Academic)
[7] Henkel M, 1999 *Conformal Invariance and Critical Phenomena* (Berlin: Springer)
[8] Henkel M, 2002 *Nucl. Phys.* B **641** 405
Picone A and Henkel M, 2004 *Nucl. Phys.* B **688** 217
[9] Henkel M and Pleimling M, 2005 *J. Phys.: Condens. Matter* **17** S1899
[10] Bouchaud J P and Bray A J, 2000 *Soft and Fragile Matter* ed M E Cates and M R Evans (Bristol: Institute of Physics Publishing)
Bray A J, 1994 *Adv. Phys.* **43** 357
[11] de Dominicis C and Peliti L, 1978 *Phys. Rev.* B **18** 353
[12] Janssen H K, 1992 *From Phase Transitions to Chaos* ed G Györgyi *et al* (Singapore: World Scientific) p 68
[13] Fisher D S and Huse D A, 1988 *Phys. Rev.* B **38** 373
Huse A D, 1989 *Phys. Rev.* B **40** 304
[14] Picone A and Henkel M, 2002 *J. Phys. A: Math. Gen.* **35** 5575
[15] Henkel M, Pleimling M, Godreche C and Luck J-M, 2001 *Phys. Rev. Lett.* **87** 265701
[16] Henkel M, Enss T and Pleimling M, 2006 *J. Phys. A: Math. Gen.* **39** L589
[17] Henkel M, 2006 *Preprint* cond-mat/0609672
[18] Berthier L, Barrat J-L and Kurchan J, 1999 *Eur. Phys. J.* B **11** 635
[19] Mazenko G F, 2004 *Phys. Rev.* E **69** 016114
[20] Godrèche C and Luck J-M, 2000 *J. Phys. A: Math. Gen.* **33** 9141
Lippiello E and Zannetti M, 2000 *Phys. Rev.* E **61** 3369
Henkel M and Schütz G M, 2004 *J. Phys. A: Math. Gen.* **37** 591
[21] Mayer P, Léonard S, Berthier L, Garrahan J P and Sollich P, 2006 *Phys. Rev. Lett.* **96** 030602
[22] Mayer P, 2004 *PhD Thesis* (London: King's college)
[23] Pleimling M and Gambassi A, 2005 *Phys. Rev.* B **71** 180401(R)
[24] Calabrese P and Gambassi A, 2002 *Phys. Rev.* E **66** 066101
[25] Calabrese P and Gambassi A, 2005 *J. Phys. A: Math. Gen.* **38** R133
[26] Hinrichsen H, 2006 *J. Stat. Mech.* L06001
[27] Menyhárd N, 1994 *J. Phys. A: Math. Gen.* **27** 6139
[28] Glauber R J, 1963 *J. Math. Phys.* **4** 191
[29] Grassberger P, Krause F and von der Twer T, 1984 *J. Phys. A: Math. Gen.* **17** L105
[30] Takayasu H and Tretyakov A Yu, 1992 *Phys. Rev. Lett.* **68** 3060
[31] Cardy J L and Täuber U C, 1996 *Phys. Rev. Lett.* **77** 4780
[32] Canet L, Chaté H, Delamotte B, Dornic I and Muñoz M A, 2005 *Phys. Rev. Lett.* **95** 100601
[33] Menyhárd N and Ódor G, 1995 *J. Phys. A: Math. Gen.* **28** 4505
[34] Menyhárd N and Ódor G, 1996 *J. Phys. A: Math. Gen.* **29** 7739
[35] Menyhárd N and Ódor G, 1997 *J. Phys. A: Math. Gen.* **30** 8515
[36] Menyhárd N and Odor G, 1998 *J. Phys. A: Math. Gen.* **31** 6771

J. Stat. Mech. (2006) L11002