Multispecies annihilating random walk transition at zero branching rate: Cluster scaling behavior in a spin model

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Numerical and theoretical studies of a one-dimensional spin model with locally broken spin symmetry are presented. The multispecies annihilating random walk transition found at zero branching rate previously is investigated now concerning the cluster behavior of the underlying spins. Generic power-law behaviors are found, besides the phase transition point, also in the active phase with fulfillment of the hyperscaling law. On the other hand scaling laws connecting bulk and cluster exponents are broken—a possibility in no contradiction with basic scaling assumptions because of the missing absorbing phase.

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I. INTRODUCTION

The study of nonequilibrium model systems has attracted great attention in recent years. A variety of phase transitions has been found characterized by critical exponents, both static and dynamic. Of special interest are transitions from a fluctuating active state into an absorbing one. A wide range of models with transitions into absorbing states was found to belong to the directed percolation (DP) universality class [1]. Another universality class of interest is the so-called parity conserving (PC) class [2–5]. The mostly studied particle model in this class is branching annihilating random walk (BARW) in one dimension (1D) with an even number of offsprings ($2A\rightarrow 0$, $A\rightarrow 3A$, in the simplest case). The first example of model systems exhibiting PC-type transition was given, however, in two 1D cellular automata by Grassberger [6]. The prototype *spin* model for PC-type phase transitions was proposed by one of the authors [7] by introducing a class of nonequilibrium kinetic Ising models (NEKIM) with combined spin-flip dynamics [8] at zero temperature $T=0$ and Kawasaki spin-exchange kinetics [9] at $T=\infty$.

Transitions between active and absorbing phases have been, however, mostly studied in particle-type models. The N-BARW2 model is a classical stochastic system of N types of particles with branching annihilating random walk. For $N>1$, N types of particles A_i perform diffusion, pairwise annihilation of the same species, and branching $A_i\rightarrow A_i+2A_j$ with rate σ for $i=j$ and with rate $\sigma'/(N-1)$ for $i\neq j$. According to field theory [10] in this model the rate σ flows to zero under coarse-graining renormalization which implies that the model is always active except for the annihilation fixed point at $\sigma'=0$. It forms a universality class, the so-called N-BARW2, different from DP and PC, with well-known bulk critical exponents in 1D.

In the NEKIM model a global asymmetry of the spins (magnetic field) is known to change the PC transition into the DP type [11,12]. The question arises what is the effect of a *local* breaking of the spin symmetry in such a spin system. The first indication in this direction has come from a work of Majumdar *et al.* [13] who studied the coarsening dynamics of a Glauber-Ising chain with strong asymmetry in the annihilation rate maximally favoring $-$ spins (MDG model). These authors found the result that while the $+$ domains still coarsen as $t^{1/2}$, the $-$ domains coarsen slightly faster as $t^{1/2}\ln(t)$. As a result at late times, the system started from a random initial state decays into a fully compact state where all spins become $-$ in a slow logarithmic way $1/\ln(t)$.

In a previous paper [14] the authors presented an asymmetric spin-model (NEKIMA) by generalizing the NEKIM model which includes as a special case the MDG model. In NEKIMA there is local spin asymmetry both in the annihilation rate (favoring $-$ spins) and the diffusionlike spin-flip rate (favoring $+$ spins) and thus acting oppositely. Global scaling properties of the model have been investigated numerically as well as using cluster mean-field (MF) approximation. The N-BARW2 transition, for which no spin model had been known previously, was found at zero spin-exchange rate. In the present paper we further investigate this model at such parameter values for which in the original NEKIM model PC-type transition takes place. In the plane of the spin-asymmetry parameter and kink-branching probability we have found, by computer simulations as well as by cluster mean-field calculations, a phase diagram showing a reentrant directed-percolation line. Our main purpose, however, has been to investigate the cluster development properties (a) at and in the vicinity of the N-BARW2 line and (b) in the rest of the parameter space considered. For the mean population size $n(t)\sim t^{\eta}$, for the mean square spreading of spins $R^2(t)\sim t^z$, and for the survival probability $P(t)\sim t^{-\delta}$ generic scaling behavior has been found via computer simulations in (almost) the whole plane of the phase diagram with fulfillment of the hyperscaling law. Upon crossing the line of zero branching rate (where the phase transition takes place), however, dynamic scaling is found to be violated concerning laws connecting bulk exponents and cluster ones. We trace back such a possibility to the circumstance that the absorbing phase is missing by the N-BARW2 transition.

II. THE MODEL AND PREVIOUS RESULTS

The general form of the Glauber spin-flip transition rate in one dimension for spin s_i sitting at site i is [8] ($s_i=\pm 1$):

$$w(s_i,s_{i-1},s_{i+1})=\frac{\Gamma}{2}(1+\tilde{\delta}s_{i-1}s_{i+1})\left[1-\frac{1}{2}s_i(s_{i-1}+s_{i+1})\right] \tag{1}$$

at zero temperature. (Usually the Glauber model is understood as the special case $\tilde{\delta}=0$, $\Gamma=1$.)

The kink $\to 3$ kink processes are introduced via the exchange rate

$$w_{ex}(s_i,s_{i+1})=\frac{p_{ex}}{2}(1-s_is_{i+1}). \tag{2}$$

This model (called NEKIM), for negative values of $\tilde{\delta}$ in Eq. (1) shows a line of PC transitions in the plane of the parameters $(p_{ex},\tilde{\delta})$ [7]. In NEKIMA [14] the authors have extended the above model by introducing local symmetry breaking in the spin-flip rates of the $+$ and $-$ spins as follows. Concerning the annihilation rates the prescription in Ref. [13] is followed:

$$w(+;--)=1, \quad w(-;++)=0, \tag{3}$$

while further spin-symmetry breaking is introduced in the diffusion part of the Glauber transition rate as follows. In calculating the transition rates

$$p\equiv w(-;+-)=w(-;-+)=\Gamma/2(1-\tilde{\delta}) \tag{4}$$

the Glauber form, Eq. (1), is used unchanged, while $w(+;+-)$ and $w(+;-+)$ are allowed to take smaller values:

$$p_+\equiv w(+;+-)=w(+;-+)\leqslant p. \tag{5}$$

In this way, by locally favoring the $+$ spins, the effect of the other dynamically induced fields arising from the prescription [Eq. (3)] is counterbalanced. The spin-exchange part of the NEKIM model remains unchanged, Eq. (2). It is worth mentioning that the lifting of the strong restriction in Eq. (3) together with applying spin anisotropy $p_+<\Gamma/2(1-\delta)$ has the same effect as a global magnetic field favoring $+$ spins.

For $p_{ex}=0$, the absorbing states in the extreme situation $p_+=0$, when diffusionlike spin flipping maximally favors $+$ spins, are states with single frozen $-$ spins like $+-+++-++-+++$. By increasing p_+ from zero, a slow random walk of these lonely $-$- spins starts and by annihilating random walk only one of them survives and performs random walk (RW) (see Fig. 1). All $+$ and all $-$ states are, of course, also absorbing.

In Ref. [14] the authors have studied the following global quantities for different values of $p_+<p$: the density of kinks as a function of time, starting from a random initial distribution of spins for $p_{ex}=0$:

$$\rho(t)\sim t^{-\alpha} \tag{6}$$

and its asymptotic values for finite but small values of p_{ex}

$$\rho_\infty(p_{ex})\sim p_{ex}^{\beta}. \tag{7}$$

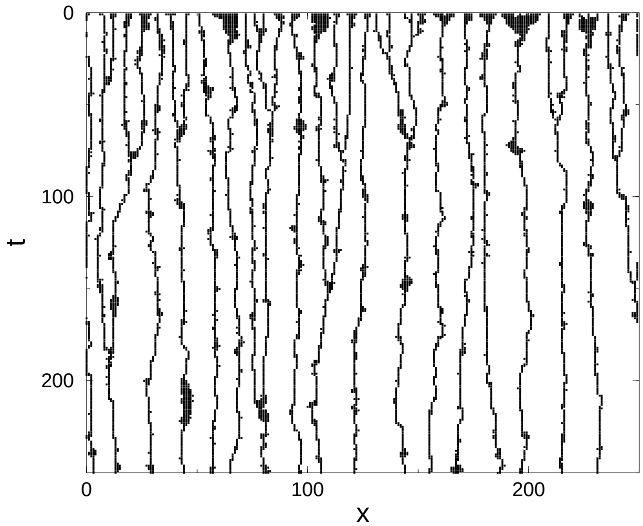

FIG. 1. Space-time development of $+$ (white) and $-$ (black) spins evolving from a random initial state for $p_+=0.1$, $p_{ex}=0$. Throughout the whole paper t is measured in units of Monte Carlo sweeps.

The results obtained within error of simulations, $\alpha=0.5$ and $\beta=1.0$, pointed to the presence of an N-BARW2 transition. Finite size scaling behavior was also examined to find the other two bulk exponents, ξ, the correlation length and τ, the characteristic time:

$$\xi\sim p_{ex}^{-\nu_\perp}, \quad \tau\sim\xi^{Z}, \tag{8}$$

where Z is the dynamical critical exponent. The expectation for an N-BARW2 transition at zero branching rate was justified by the values: $\nu_\perp=1.0$ and $Z=2.0$, which were found within error of simulations. We also found the expected phase diagram of a line of DP transitions in the (p_+,p_{ex}) plane (instead of the PC line of NEKIM).

III. CLUSTER BEHAVIOR AT AND BELOW THE N-BARW2 TRANSITION

Spreading from a localized source at criticality is usually described by the following three quantities:

$$P(t)\sim t^{-\delta}, \quad n(t)\sim t^{\eta}, \quad R^2(t)\sim t^{z}, \tag{9}$$

where $n(t)$ denotes the mean population size, $R^2(t)$ is the mean square spreading of particles (here spins) about the origin, and $P(t)$ is the survival probability. In most cases these quantities are defined for particles, in the present case, however, like for studying compact directed percolation of an Ising chain [15], they will be used for spins.

In the active phase the survival probability defines a further useful critical exponent β' ($\nu_\|=Z\nu_\perp$)

$$P\sim t^{-\delta}g(p_{ex}t^{1/\nu_\|}) \tag{10}$$

as

$$P_\infty\sim p_{ex}{}^{\beta'}, \quad \beta'=\nu_\|\delta. \tag{11}$$

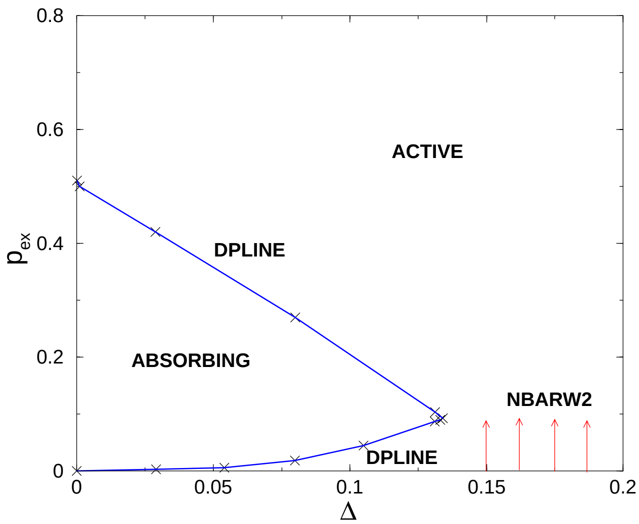

FIG. 2. Phase diagram of the NEKIMA model for $\delta = -0.565$, $\Gamma = 0.5$. The absorbing phase is fully $-$.

In the present case the following parameter values of NEKIMA were used in the simulations: $\Gamma = 0.5$, $\tilde{\delta} = -0.565$ ($p = 0.39125$). The phase diagram in the ($\Delta \equiv (p - p_+)/p, p_{ex}$) plane is shown in Fig. 2. The origin (0,0) will be called "MDG point" as at this point $p_+ = p$ and the model is the same as treated in Ref. [13] (though the values of $\tilde{\delta}$ and Γ are different). The line $p_{ex} = 0$ is a line of compactness, as will be discussed in the following section. Also other details of the phase diagram will be explained later.

In NEKIMA $+$ and $-$ spins are not symmetric, therefore we have investigated two kinds of clusters. Namely, the development of the $-$ cluster seed was started from a wholly $+$ environment at $t=0$: $++++++++ - ++++++++$, while the $+$ cluster from a sea of $-$ spins: - - - - - - $+$ - - - - - - . We will call them $-$ cluster and $+$ cluster, respectively. The simulations have been performed with several values of p_+ and p_{ex}; for $t_{max} = 5 \times 10^3$ Monte Carlo (MC) steps and for averages over 10^4 samples. The local slopes

$$-\delta(t) = \frac{\ln[P(t)/P(t/m)]}{\ln m} \qquad (12)$$

[and similarly for $\eta(t)$ and $z(t)$] as a function of $1/t$ are plotted, as usual in case of simulations for critically behaving quantities. In Eq. (12) $m > 1$ is an arbitrary factor which we took to be equal to 5. The results obtained in different regions of the phase diagram, Fig. 2, are summarized on Tables I and II. As it is apparent from Table I, the $+$ cluster does not change its exponents by crossing the $p_{ex} = 0$ line. The $-$ cluster's exponents, however, change abruptly.

TABLE I. Cluster critical exponents at and near the N-BARW2 transition point. The hyperscaling law $\eta + \delta = z/2$ (see Sec. V) is satisfied.

Exponents	$p_{ex}=0$, $+$	$p_{ex}=0$, $-$	$p_{ex}\neq 0$, $+$	$p_{ex}\neq 0$, $-$
η	1.0	0.0	1.0	1.0
δ	0.0	0.0	0.0	0.0
z	2.0	0.0	2.0	2.0

For the case $p_+ = 0.3$ Figs. 3 and 4 show the local exponent values, [Eq. (12)] for $p_{ex} = 0$, i.e., at the N-BARW2 transition point and for $p_{ex} = 0.02$, i.e., in the active phase. Here and in most cases of our simulations the number of MC steps has been 5×10^3 with averaging over 2×10^4 different runs. In some cases, however, much longer runs have also been carried out up to 10^5 MC steps to corroborate these results, see Fig. 5.

For comparison let us recall the well-known values for the above exponents in case of the compact directed percolation point of the Domány-Kinzel cellular automaton [16]. Dickman and Tretyakov [15] have given the results in this context as follows: $\eta = 0$, $\delta = 1/2$, and $z = 1$. (The same as for the Glauber-Ising model at $\tilde{\delta} = 0$, $\Gamma = 1.0$.) It is of some interest to present the measured cluster exponents at the origin of the phase diagram, Fig. 2, which is the equivalent of the MDG point. Here we found for the $+$ cluster: $\eta = 0$, $\delta = 1/2$, $z = 1$ while for the $-$ cluster: $\eta = 1/2$, $\delta = 0$, $z = 1$ [with the same accuracy as most of our results here ($t_{max} = 5 \times 10^3$ MC steps)]. These data are summarized in Table III. Because of the relatively low upper limit in time of most of our simulations as given above, the possibility of the presence of a $\ln(t)$ correction at the MDG point cannot be excluded.

IV. BREAKING OF A SCALING LAW

According to the preceding section the result for the critical exponent of the mean square distance of spreading from the origin, z, is equal to 2.0 within error of numerical simulations. For the dynamical critical exponent the value $Z = 2.0$ was obtained, in the whole regime (p_+ values) of the N-BARW2 transition.

On the other hand, at BARW-type transitions, such as DP and PC transitions, the following scaling law connects the above two critical exponents:

TABLE II. Cluster critical exponents at and near the DP line. For abbreviations see Fig. 2. The hyperscaling law, valid for DP transitions, $\eta + 2\delta = z/2$ (see Sec. V) is satisfied.

Exponents	On DP line $+$	On DP line $-$	Absorbing phase $+$	Absorbing phase $-$	Active phase $+$	Active phase $-$
η	0.31	1.0	Exponential	1.0	1.0	1.0
δ	0.16	0.0	Exponential	0.0	0.0	0.0
z	1.26	2.0	Exponential	2.0	2.0	2.0

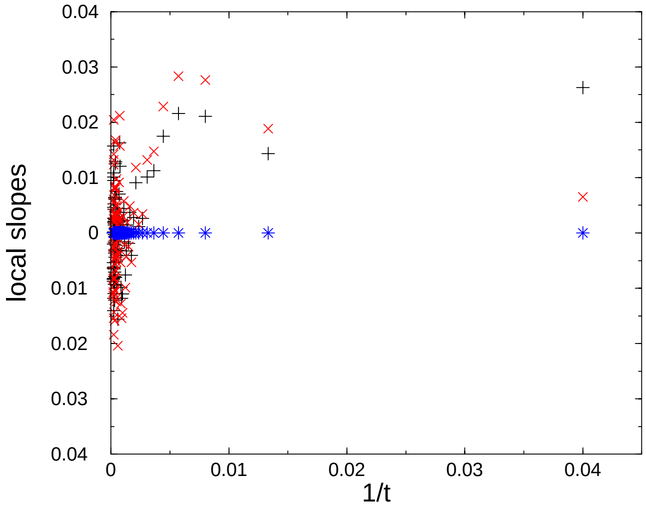

FIG. 3. Cluster exponents for − cluster at $p_+=0.3$, $p_{ex}=0$. Number of MC steps: 10^5, number of averages: 10^3. + correspond to η, × to $z/2$, and * to δ.

$$z=2/Z. \tag{13}$$

This relation is usually quoted as a consequence of dynamical scaling. Using the above cited results, however, Eq. (13) is broken. The possibility of breaking this scaling law is actually due to the circumstance that the N-BARW2 transition point lies at the zero value of the branching probability, $p_{ex}=0$, and there is no absorbing phase with exponentially decreasing space and time dependences. To support this point let us recall the way Mendes *et al.* [17] derived relation (13).

They started from the general expression [18] for the density of particles (kinks) at space point r in the absorbing phase $\Delta<0$ (here Δ denotes the deviation from the critical point) and at large fixed value of t (for $d=1$)

$$\rho(r,t)=t^{\eta-z/2}F(r^2/t^z,\Delta t^{1/\nu_\parallel}). \tag{14}$$

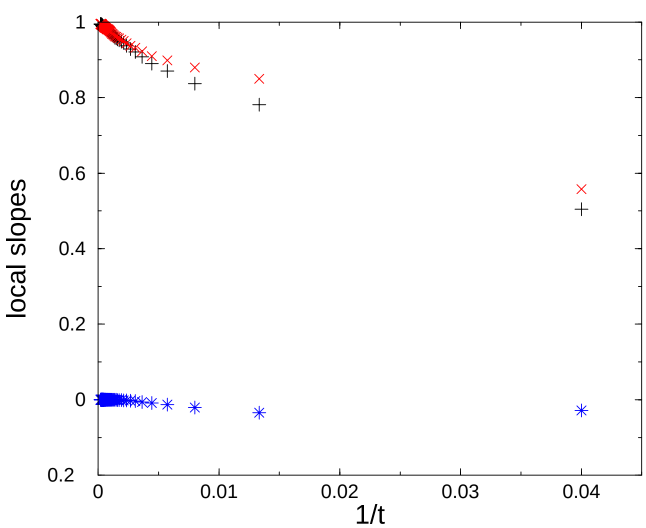

FIG. 4. Cluster exponents for − cluster at $p_+=0.3$, $p_{ex}=0$. Number of MC steps: 10^5, number of averages: 10^3. + correspond to η, × to $z/2$, and * to δ.

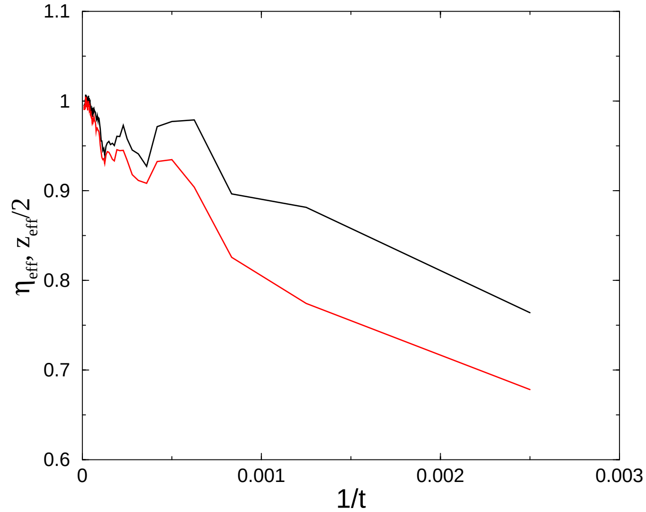

FIG. 5. Cluster spreading exponents for + cluster at parameter values $p_+=0.3$, $p_{ex}=0.02$ ($\delta=-0.565,\Gamma=0.5$). Number of MC steps: 10^5, number of averages: 10^3. Upper curve: η_{eff}, lower curve: $z_{eff}/2$.

In the absorbing phase the function $\rho(r,t)$ is expected to decrease exponentially as $\rho(r,t)\sim\exp-r/\xi$, where $\xi\sim\Delta^{-\nu_\perp}$. This form implies for $F(u,v)$ (with $v<0$) the form

$$F(u,v)\to\exp(-C\sqrt{u}|v|^{\nu_\perp}), \tag{15}$$

where $C>0$ is constant. For ξ to be time independent the scaling law is required:

$$z=\frac{2\nu_\perp}{\nu_\parallel}=\frac{2}{Z}. \tag{16}$$

This scaling law is not fulfilled in the presently discussed model. Moreover, the bulk quantity, the time dependent kink density

$$\rho(t)\sim t^{-\alpha}, \tag{17}$$

and the expression obtainable from Eq. (14)

$$\rho(t)\sim t^{\eta-z/2} \tag{18}$$

are also in conflict; namely, while all the simulations have resulted in $\alpha=0.5$ within error and this is in agreement with the scaling law $\alpha=\beta/\nu_\parallel$, according to the values given in

TABLE III. Cluster critical exponents in case of the Glauber-Ising and MDG parameter values. The hyperscaling law $\eta+\delta=z/2$ is satisfied.

Exponents	Glauber-Ising, +	Glauber-Ising, −	MDG, +	MDG, −
η	0	0	0.0	0.5
δ	1/2	1/2	0.5	0.0
z	1	1	1.0	1.0

Table I the exponent in Eq. (18) is zero, again within the error of simulations. (It is to be noted that this conflict is no more present concerning the exponent values at $p_{ex}\neq 0$, where $\alpha=0$ and $\eta-z/2=0$, as well.) The apparent contradiction, however, is resolved again by recalling that cluster exponents and bulk exponents are allowed to be not connected by a scaling law.

V. HYPERSCALING

The generalized hyperscaling law [18] was developed for systems with multiple absorbing configurations and reads

$$2\left(1+\frac{\beta}{\beta'}\right)\delta+2\eta=dz, \tag{19}$$

where β' is defined for the active phase, Eq. (11).

The derivation of Eq. (19) goes along the following lines. It starts with Eq. (14) for $\rho(r,t)$ and with the expressions

$$P(t)\sim t^{-\delta}\Phi(\Delta t^{1/\nu_{\parallel}}), \tag{20}$$

$$P_{\infty}\sim\Delta^{\beta'}, \quad \beta'=\delta\nu_{\parallel} \tag{21}$$

for the survival probability. Since the stationary distribution is unique

$$\rho(x,t)\to P_{\infty}\Delta^{\beta}\sim\Delta^{\beta+\beta'} \tag{22}$$

as $t\to\infty$. Hence $F(0,y)\sim y^{\beta+\beta'}$ which entails Eq. (19).

In case of the DP transition (along the DP line of Fig. 2) $\beta'=\beta$ as it is well known, and thus Eq. (19) gives

$$2\delta+\eta=z/2. \tag{23}$$

For the N-BARW2 transition, however, Eq. (19) does not apply as Eq. (14), according to the preceding section, is not an appropriate starting point.

To deduce the hyperscaling law valid for this case there are several possible ways of arguing. It is possible to enlarge the parameter space of our model: we can think of a third direction in the parameter space, approaching from where the transition turns out to be of first order. For this aim one can introduce a "magnetic field" into the system by changing the annihilation probability as $w(+;--)=1-h$. In this direction $\beta_h=0$ and thus Eq. (19) gives ($d=1$)

$$\eta+\delta=z/2. \tag{24}$$

This law is satisfied for all the clusters investigated, including those at the MDG point. Even for $p_{ex}\neq 0$ we can still think of each point as being a first-order transition point with $\beta=0$ in the h direction and the same considerations apply as above. Thus on the basis of the results presented now, the conclusion to be drawn is that hyperscaling is generically satisfied in the whole N-BARW2 phase of the NEKIMA model.

Looking at the problem from a different point of view, however, it is really not necessary to introduce the above auxiliary magnetic field; namely, one can simply make the observation that all the clusters investigated on the $p_{ex}=0$ line are compact and from this fact Eq. (24) follows for the hyperscaling law [15].

Equation (24) is known as the hyperscaling law for compact clusters. By definition $\delta+\eta$ is the exponent which characterizes the average population in surviving trials and the radius of such a cluster grows as $R_t\sim t^{z/2}$. $\delta+\eta=dz/2$ is simply the scaling law for the volume of a d-dimensional sphere of radius R_t [15].

As a matter of fact, the $-$ clusters are compact with $\delta=0$ also at $p_{ex}\neq 0$, and even in the DP region of Fig. 2. This is not true, however, for the $+$ cluster in the DP region, which follows normal DP-cluster behavior (see Table II) with the corresponding DP hyperscaling law (23). (It is worth noting that whenever $\delta=0$, and this fails only for the $+$ cluster in the DP region, the CDP and DP hyperscaling laws do not differ.)

VI. REENTRANT PHASE DIAGRAM, CLUSTER MF CALCULATIONS

In the original NEKIM model at $\tilde{\delta}\geqslant 0$ no transition occurs, while for negative values of this parameter PC transition takes place. The spin asymmetry of NEKIMA changes the character of the transition into DP and this appears also for $\tilde{\delta}\geqslant 0$. Here we have chosen for our simulations and for the cluster mean-field approximation calculations a fixed negative value of $\tilde{\delta}$. Our aim has been to explore some possible reminiscence of the PC transition. At the chosen parameter values $\tilde{\delta}=-0.565,\Gamma=0.5$ in NEKIM the PC transition occurs at $p_{ex}=0.12$. Turning to NEKIMA, at the same values of $\tilde{\delta},\Gamma$ our simulations show that the transition point (which is DP, of course) shifts to $p_{ex}=0.51$. The absorbing phase below this point is all $-$. For letting $p_+<p$ the DP line starts tangentially upon increasing p_{ex} from 0 and exhibits a reentrant property. It ends up at $p_{ex}=0.51$ tangentially. The regression takes place at $p_{ex}\approx 0.12$, see Fig. 2, most probably a remnant of the transition point of the corresponding PC transition. This turning point, however, is also of DP character as can be expected.

Dynamical cluster mean-field approximations have been introduced for nonequilibrium models by Refs. [19,20]. The master equations for $N=1$–7 block probabilities were set up as

$$\frac{\partial P_N(\{s_i\})}{\partial t}=f(P_N(\{s_i\})), \tag{25}$$

where site variables may take values $s_i=\pm 1$. Taking into account spatial reflection symmetries of $P_N(\{s_i\})$ this involves 72 independent variables in case of $N=7$. The equations were solved numerically for the $\partial P_N(\{s_i\})/\partial t=0$ steady state condition, for different p_{ex} and p_+ values and the $\rho_k(\infty)$ kink density was expressed by $P_N(\{s_i\})$. The reentrant behavior could not be observed for $N<6$ clusters.

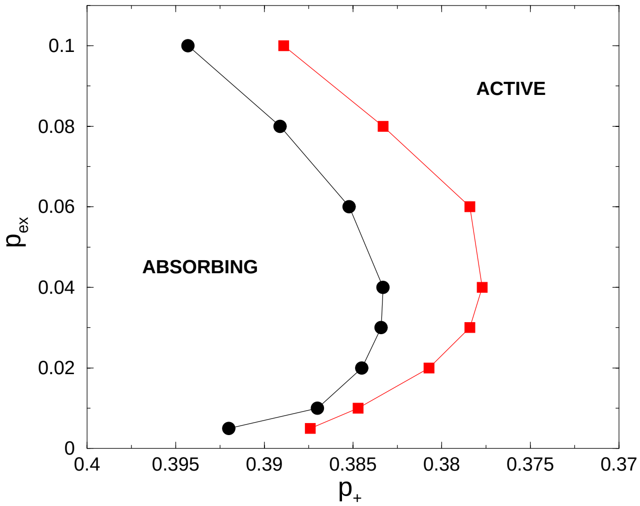

FIG. 6. Steady state density in $N=6$ (bullets) and $N=7$ (boxes) level approximation. Lines connecting symbols are shown for guidance of eye only.

The results for $N=6,7$ are shown in Fig. 6. A slow shift towards lower p_+ values, which agree with the simulations, can be observed.

VII. DISCUSSION

We have investigated a one-dimensional NEKIMA exhibiting strong spin asymmetry. In the plane of two of the parameters of NEKIMA (the kink-branching parameter and a spin-asymmetry parameter) the phase diagram is as follows: besides a reentrant DP line the NBARW-2 transition occurs at zero branching rate. Due to the asymmetries, + and − spin clusters behave differently. By investigating their development we conclude that generic power-law behavior characterizes the cluster behavior at and in the vicinity of the NBARW-2 transition.

The critical cluster exponents obtained satisfy the constraints on critical exponents in general: (1) $\delta \geqslant 0$ and (2) $1 \leqslant z \leqslant 2$. The critical exponent δ has been found to be zero. The hyperscaling law is satisfied in the form known for compact directed percolation, and indeed, the N-BARW2 clusters are compact.

In a different problem Cafiero *et al.* [21] have reported cluster exponents similar to the ones found here. These authors studied how disorder affects the critical behavior of DP-like systems. Already in the 1980s Noest [22] showed that quenched disorder changes their behavior in $d<4$ and demonstrated that when $d=1$ a generic scale invariance can be observed. In Ref. [21] it was shown that deep in the active phase $\eta=1$, $\delta=0$, and $z=2$ for the model they considered. As we have found also generic scale invariance and the same exponents, in the active phase of our model and even in the region which is the active phase of the DP line of our phase diagram, the question arises whether the similarity is fortuitous or not. Whether the slowly diffusing − clusters of NEKIMA distributed randomly in the x direction can play a role similar to quenched impurities, e.g., in the original NEKIM model, is a question for future investigations.

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[1] For a review, see J. Marro and R. Dickman, *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge University Press, Cambridge, 1996); H. Hinrichsen, Adv. Phys. **49**, 815 (2000); G. Ódor, Rev. Mod. Phys. (to be published), e-print cond-mat/0205644.
[2] I. Jensen, Phys. Rev. E **50**, 3623 (1994).
[3] D. Zhong and D. ben-Avraham, Phys. Lett. A **209**, 333 (1995).
[4] M.H. Kim and H. Park, Phys. Rev. Lett. **73**, 2579 (1994).
[5] H. Park, M.H. Kim, and H. Park, Phys. Rev. E **52**, 5664 (1995).
[6] P. Grassberger, F. Krause, and T. von der Twer, J. Phys. A **17**, L105 (1984); P. Grassberger, *ibid.* **22**, L1103 (1989).
[7] N. Menyhárd, J. Phys. A **27**, 6139 (1994).
[8] R.J. Glauber, J. Math. Phys. **4**, 191 (1963).
[9] See, e.g., K. Kawasaki, *Phase Transitions and Critical Phenomena*, edited by C. Domb and M.S. Green (Academic Press, New York, 1972), Vol. 2, p. 443.
[10] J. Cardy and U.C. Täuber, Phys. Rev. Lett. **77**, 4780 (1996); J. Stat. Phys. **90**, 1 (1998).
[11] H. Park and H. Park, Physica A **221**, 97 (1995).
[12] N. Menyhárd and G. Ódor, J. Phys. A **29**, 7739 (1996).
[13] S.N. Majumdar, D.S. Dean, and P. Grassberger, Phys. Rev. Lett. **86**, 2301 (2001).
[14] N. Menyhárd and G. Ódor, Phys. Rev. E **66**, 016127 (2002).
[15] R. Dickman and Y. Tretyakov, Phys. Rev. E **52**, 3218 (1995).
[16] E. Domány and W. Kinzel, Phys. Rev. Lett. **53**, 311 (1984).
[17] J.F.F. Mendes, R. Dickman, M. Henkel, and M. Marques, J. Phys. A **27**, 3019 (1994).
[18] P. Grassberger and A. de la Torre, Ann. Phys. (N.Y.) **122**, 373 (1979).
[19] H.A. Gutowitz, J.D. Victor, and B.W. Knight, Physica D **28**, 18 (1987).
[20] R. Dickman, Phys. Rev. A **38**, 2588 (1988).
[21] R. Cafiero, A. Gabrielli, and M.A. Muñoz, Phys. Rev. E **57**, 5060 (1998).
[22] A.J. Noest, Phys. Rev. Lett. **57**, 90 (1986); Phys. Rev. B **38**, 2715 (1988).